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EDITORIAL

The Unit-pole Definition of Magnetic Field Strength

HAVING defined unit pole as a pole of such strength that if placed 1 cm away from a similar pole in air it experiences a force of 1 dyne, one then generally proceeds to define the strength of the magnetic field at any point in space as the force in dynes experienced by unit pole if placed at the point. This suggests that H is a measurable magnitude. In reviewing Lanchester's "Theory of Dimensions," in *Nature*, of January 9th, 1937, we showed, however, that E and B are measurable characteristics of the electric and magnetic fields, the former by the force on a stationary charge and the latter by the force on a current or moving charge, irrespective of the medium, whereas D and H are not directly measurable concepts.

The displacement D is calculable at any point by dividing a charge by an area; and similarly the magnetizing force H is calculable by dividing a current—or a number of ampere-turns—by a length. From this point of view H or the ampere-turns per cm must be regarded as a localized cause producing at every point a magnetic induction B depending on the medium. Similarly D appears as a localized phenomenon related at every point to a condition of space designated by E, the relationship depending on the medium. We pointed out the parallel phenomena in the theory of elasticity where the localized cause is called the stress at the point and the effect is called strain, and we quoted from Prof. Southwell's "Theory of Elasticity" that "stress has never been measured directly, and we can assert with some confidence that it never will." Hence we have three non-measurable magnitudes, viz., stress = force/area, H = current/length, and D = quantity of electricity/area, and three related measurable magnitudes, viz., strain, B, and E. The latter can be determined by measurements made at the point; the former have to be calculated on certain assumptions as to the distribution of mechanical force or electric charge over an area or of the ampere-turns over the magnetic path-length.

How is this to be reconciled with the determination of H at any point by measuring the force on unit pole placed there? So long as we confine our attention to experiments in air or any medium of unit permeability one might be tempted to regard it as immaterial whether we regard the measurement as one of H or B. In a medium of permeability other than unity the force on unit pole raises some interesting questions, and is well worth considering. The first question is, what happens to the magnet of unit pole strength when it is immersed in a magnetic medium?

It is pointless to say that one must assume it to remain of unit strength, because unit strength has only been defined in air. Two unit poles certainly do not retain the property of repelling each other with a force of 1 dyne

when placed 1 cm apart in the new medium. One is forced to assume that the magnetic flux of the magnet remains unchanged, but this is introducing a new concept and undermining the fundamental character of the unit pole. As we have pointed out on a previous occasion, if the magnet of unit pole strength is not a permanent magnet but an empty solenoid of which the current is adjusted until it repels a similar pole with a force of one dyne at a distance of one centimetre, and if two such solenoids are then immersed in a magnetic medium the force between the poles is increased to μ times its value in air. This follows at once from the fact that the value of B is everywhere increased in this ratio since the current has been kept constant. If, however, the current in each solenoid is reduced in the ratio $1/\mu$, B will have its original value and the force on the pole will be reduced in the same ratio as the current, that is $1/\mu$. This agrees with the formula usually given for the force between two poles, viz., $f = m_1 m_2/\mu d^2$, which bears out what we said above as to the unit pole being based on the assumption of a constant magnetic flux.

One can make a solenoid, however, in which the flux remains constant whatever the external medium without any adjustment of current, merely by winding it on a solid core and making the length very great compared with the diameter. The value of H, and therefore also of B, within such a solenoid depends only on the ampere-turns per cm and is unchanged by immersing the solenoid in a medium of high permeability. If now the medium be subjected to a magnetizing force H_x, the magnetic induction $B_x = \mu H_x$ but the forces on the solenoid will be the same as they were in air for the same value of H_x, that is to say, the solenoid appears to measure H and not B in the medium in which it is immersed. We say " appears to " because on looking into the matter more closely it is seen that we have carried out a cavity experiment. We have constructed a long narrow cavity filled with a material of unit permeability—assuming the coil to be wound on such a material—and in that cavity or on its walls we have placed a coil carrying a certain fixed current. Any force on the coil will depend on the value of B in the cavity, which is numerically equal to H in the cavity since $\mu_1 = 1$.

Consider two positions of the solenoid or cavity, one in the direction of the external field and the other in the opposite direction. In one case the solenoid field H_s will be increased to $H_s + H_x$ and in the other it will be decreased to $H_s - H_x$. If the solenoid has a cross-sectional area A and a length l, and carries a current of I e.m. units, the energy supplied to the circuit on reversing the coil will be $2AH_xTI$ ergs, which, putting $TI = H_s l/4\pi$, is equal to $H_s H_x Al/2\pi$. This will be equal to the work done in turning the solenoid from one position to the other, and if the pole strength is m this should be equal to $2lmH_x$ which, putting $m = H_s A/4\pi$, is equal to $H_s H_x Al/2\pi$. Hence the force on the pole is proportional to H_x and is independent of the permeability of the surrounding medium, but, as we saw in the preceding editorial, this idea of the torque on the solenoid being due to forces acting on poles near the ends gives an erroneous picture of the nature of the forces acting on the coil.

A very interesting problem is the calculation of the torque on the above solenoid when placed at right-angles to the direction of the field. At first sight it gives a surprising result. For the sake of simplicity we assume the solenoid to be inside a glass tube filled with some material of unit permeability and the outer medium to have a very high permeability. We assume that the length is very great compared with the diameter and, to avoid trouble with end effects, we assume that the tube is longer than the solenoid. Fig. 1 shows a section of the tube.

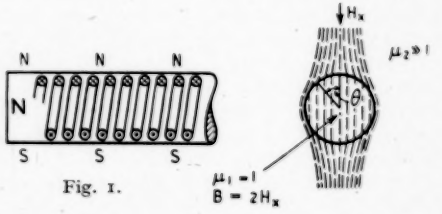

Fig. 1.

It has been shown* that in such a case the radial magnetic force H_r on the inner surface is equal to

$$\frac{2\mu_2}{\mu_2 + \mu_1} H_x \cos\theta$$

where H_x is the applied uniform value of H in the external medium apart from local distortion due to the presence of the tube, μ_1 and μ_2 the internal and external permeabilities, and θ the angle between the radius

* B. Hague. *Journ. Instn. Elec. Engrs.*, Vol. 64, p. 476, April 1926.

considered and the direction of H_x. In our case $\mu_1 = 1$ and $H_r = \frac{2\mu_2}{\mu_2 + 1} H_x \cos\theta$. If μ_2 is very great this approximates to $2H_x \cos\theta$, that is to say, the uniform field in the tube is equal to $2H_x$. Hence we arrive at the surprising result that the solenoid is situated in a uniform magnetic field having twice the magnetizing force of the external uniform field in which we have placed it, and that the resulting torque on the solenoid will be twice the value corresponding to H_x. This is true of the solenoid if free to move within the glass tube, but it is not true of the tube and solenoid regarded as a unit, because half the torque on the solenoid is counterbalanced by the reaction on the polarity at the interface between the two media, and only the other half is externally measurable. Hence, although the actual force on the solenoid, as on any current-carrying conductor, is proportional to the magnetic induction B in which it is situated, the arrangement under discussion gives a measure of the magnetic force H_x in the permeable medium in which it is placed. It is really a cavity experiment.

We propose now to prove that half the torque on the actual solenoid is counterbalanced by the polarity at the interface.

When magnetic flux passes normally across the interface from a medium of permeability μ to another of unit permeability, B undergoes no change, but H is increased from B/μ to B. This change of H can be regarded as due to polarity of the interface, and if m_1 is the strength of pole per square centimetre, $4\pi m_1 = B(1 - 1/\mu)$. To an observer inside the tube in Fig. 1 the upper surface would appear to be a north pole and the lower surface a south pole. If the tube were filled with the external medium of very high permeability, the field would be everywhere uniform and H within the tube would be equal to H_x; but on decreasing the permeability within the tube to unity, H within the tube increases to $2H_x$ although the external magnetizing agent has not changed. This increase in the magnetizing force is due to the polarity of the interface. That this is so can be proved as follows: if μ_2 is very great $4\pi m_1 = B(1 - 1/\mu_2) \approx B$ where B is the radial flux density at the interface, i.e.

$$B = H_r = 2H_x \cos\theta;$$

hence $m_1 = 2H_x \cos\theta / 4\pi$ per cm².

If Fig. 2 represents a strip along the tube of width $r d\theta$ and of pole strength $m_1 r d\theta$ per unit length, the magnetic force at a point P at distance r from the strip is given by the formula

$$H = 2m_1 r d\theta \int_0^\infty \frac{dx}{x^2 + r^2} \times \frac{r}{\sqrt{x^2 + r^2}} = 2m_1 \, d\theta$$

assuming the strip to be very long compared with r.* Hence substituting for m_1 we have for the value of H at the centre of the tube, due to the top and bottom half-cylinders,

Fig. 2.

$$H = 4 \int_{-\pi/2}^{+\pi/2} \frac{2H_x \cos^2\theta}{4\pi} d\theta = \frac{2H_x}{\pi} \int_{-\pi/2}^{+\pi/2} \cos^2\theta \, d\theta = H_x$$

Hence, assuming the external medium to have an infinitely great permeability, the field strength $2H_x$ within the tube is made up of two equal parts, one due to the external magnetizing agent and the other to the polarity on the walls of the tube. For simplicity we have assumed the medium within the tube to have unit permeability; for any other value of μ_1, the transverse magnetic force in the tube will still be made up of two components, one equal to H_x as before, but the other less than H_x. The former will produce an induction $B = \mu_1 H_x$ and since the current to produce unit pole strength in the solenoid has been reduced in the ratio $1/\mu_1$, the externally measurable torque remains unchanged. The internal torque between the tube and coil is reduced, and of course vanishes entirely when μ_1 is made equal to μ_2 of the external medium.

The exact nature of the internal forces between the solenoid and the polarity of the tube walls is easily seen when one remembers that in a very long solenoid of small diameter the flux is almost constant except near the ends, where it rapidly drops—to a half at the end and then to zero at a short distance beyond the end. Hence the whole flux Φ of the solenoid leaves radially at the end, and if, for simplicity, we assume it to leave uniformly over a short length s, the radial

* $\int \frac{dx}{(x^2 + r^2)^{3/2}} = \frac{1}{r^2} \cdot \frac{x}{\sqrt{(x^2 + r^2)}}$

flux density will be $\Phi/2\pi rs$ which, putting $\Phi = 4\pi ITA/l$, gives

$$H_r^1 = 2ITA/rsl$$

This is the radial field strength due to the solenoid in which the polarity m_1 per cm^2 of the walls is situated. We have seen that $m_1 = H_x \cos\theta/2\pi$. The radial force per cm^2 will be $m_1 H_r^1$ and its component in the vertical direction $m_1 H_r^1 \cos\theta$. This acts over an area of $2\pi rs$, hence the total resultant vertical force will be

$$\frac{H_x}{2\pi} \times \frac{2ITA}{rsl} \times 2\pi rs \times \text{average value of } \cos^2\theta$$
$$= H_x ITA/l$$

The torque is the product of this force and the length of the solenoid; i.e., $H_x ITA$. Since the pole strength $m = ITA/l$ this is equivalent to $H_x ml$. This, then, is the internal torque exerted mutually between the coil and the tube, and, in this case in which $\mu_1 = 1$ and μ_2 is very large, it is seen to be exactly equal to the external torque.

Although this case is interesting and simple, a much more practical case is that in which the core of the solenoid has a high permeability and the surrounding medium unit permeability. We assume that the coil is embedded in a rod of high permeability and, to avoid end effect, we assume, as before, that the rod is longer than the coil. Conditions will now be as shown in Fig. 3 with $B = 2H_x$ and $H = 2H_x/\mu_1$; the latter will be very small since the magnetizing force due to the polarity is upwards and nearly neutralizes H_x. If we had started

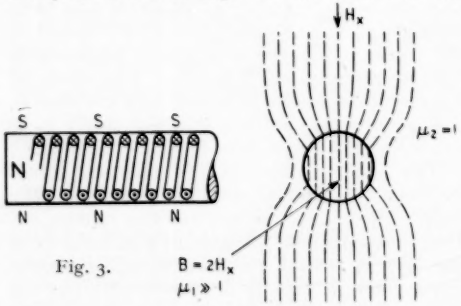

Fig. 3.

with the surrounding medium also of the same high permeability μ_1, there would have been no interface polarity, and the torque would have been

$$mlH_x = ITA\mu_1 H_x$$

exerted solely on the current carrying coil, but as the external permeability was decreased, keeping H_x and I unchanged, the external B would decrease to H_x and the internal B to $2H_x$. Since the internal B has decreased from $\mu_1 H_x$ to $2H_x$ while I has remained constant, the torque exerted on the coil must be reduced in the ratio $2/\mu_1$, that is, to a small fraction of its former value although the resultant torque on the coil and core must have remained unchanged since m and H_x are still the same. In the previous example the calculated torque on the coil was twice the expected value mlH_x, the excess being counterbalanced by the torque due to the polarity of the interface; in the present example the calculated torque on the coil is only a small fraction of the expected value, but the deficit must be made up by torque due to the interface polarity, which is, as we have seen, the reverse of the previous polarity. In the limit when μ_1 is very great, the torque on the coil is negligible and the whole torque mlH_x is due to the polarity of the interface.

Instead of considering the torque as being caused by the interaction between the polarity of the interface and the flux leaving the coil at the ends, one may regard the problem from a simpler but more fictitious point of view by picturing the flux density across the tube as the resultant of two flux densities each causing their own force on the current-carrying coils. In the first example there was $B = H_x$ due to the external agency plus $B = H_x$ due to the surface polarity, giving a double torque on the coil. In the second example there was $B = \mu_1 H_x$ due to the external agency minus $B = (\mu_1 - 2)H_x$ due to the surface polarity, exerting opposing torques on the coil with a very small resultant.

In the more general case the two opposing values of B are

$$H_x\mu_1 \text{ and } H_x\mu_1(\mu_1 - \mu_2)/(\mu_1 + \mu_2).$$

Our last example approximates closely to a permanently magnetized knitting needle. It is a rod of high permeability in which the constant magnetism is maintained, not by an embedded solenoid, but by myriads of circulating electrons.

We have seen that although such a permanently magnetized needle or a solenoid can be used to determine H, the measurement is primarily of the force on a current due to the magnetic induction B in which it is situated, the determination of H being derived by what is really a cavity experiment.

G. W. O. H.

ANOMALOUS ATTENUATION IN WAVEGUIDES*

By John Kemp

(Standard Telephones and Cables, Limited)

SUMMARY.—The puzzling phenomenon of decreasing attenuation constant with increasing frequency which occurs in a few isolated instances is here elucidated by treating the guides concerned as limiting cases of a guide of more general shape in the interior of which the waves display the normal properties characteristic of waves in guides generally. The equations of the electromagnetic field, cut-off frequency and attenuation constant describing the isolated cases are then, in like manner, deduced as limiting cases from those appropriate to a guide of general shape. The isolated cases thus lose their character of isolation and assume that of straightforward limits instead. According to the point of view developed in the paper these limiting cases imply an electromagnetic field which extends to infinity along one of the transverse co-ordinates but, being wrapped around the axis of the guide, the field is constrained to exist in finite space where it continues to display the properties characteristic of a field of infinite extent.

1. Introduction

IT is a well-known fact that when an electromagnetic wave is traversing a transmission line of any conventional type, it is always possible to find a frequency above which the attenuation constant of the wave is a steadily increasing function of frequency. This property also appertains to waves propagated through the interior of hollow metal tubes with the exception, however, of a few seemingly isolated cases. There is, for instance, the case where the *H*-wave (also known as the transverse electric or *TE*-wave) of zero order is propagated through a tube of circular cross-section. Here, as in two further cases to be dealt with below, the attenuation constant follows a course precisely opposite to that normally encountered; that is to say, in each case a frequency can be found above which the attenuation steadily *decreases* and ultimately vanishes altogether.

This striking anomaly may be accounted for satisfactorily by means of Ampère's Law of electromagnetic induction, whereby the current induced in the metal tube may be stated in terms of the tangential components of the magnetic field at the surface of the metal, the longitudinal component of the current in terms of the transverse component of the field, and the transverse component of the current in terms of the longitudinal component of the field. Now in the exceptional case of the *H*-wave of zero order in a guide of circular cross-section all lines of magnetic force are at any frequency restricted to radial planes (Fig. 1) and in consequence the transverse tangential component is always zero. There thus remains only the longitudinal tangential component and as this component—in common with the longitudinal component of all other waves in hollow metal tubes—is a steadily decreasing function of frequency, the attenuation of the wave is of necessity also a diminishing function of frequency.

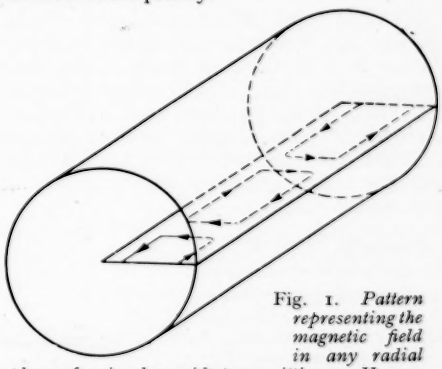

Fig. 1. *Pattern representing the magnetic field in any radial plane of a circular guide transmitting an H-wave of zero order and principal mode. If the plane is turned full circle the pattern describes a set of toroidal surfaces which completely represent the magnetic field in space.*

As an instructive alternative explanation it is here suggested that a guide of circular cross-section might be regarded as an extreme case of a guide having a cross-section of sector shape (Fig. 2). In such a guide the *H*-wave of zero order displays the normal properties characteristic of waves in guides generally and from these there emerge as straightforward limiting cases those appropriate to waves in a circular guide.

* MS. first accepted by the Editor, November 1945, in final form, February 1946.

By this procedure the appearance of an anomaly is completely eliminated. Moreover, by extending the analysis to guides encompassed by two coaxial circular cylinders and two radial planes (Fig. 3) the properties

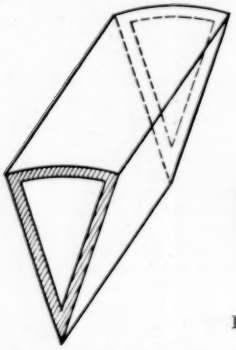

Fig. 2. *Guide of sector-shaped cross-section.*

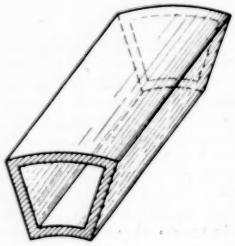

Fig. 3. *Guide composed of portions of coaxial circular cylinders and radial planes.*

of H-waves of zero order may be stated in a general form which includes as particular cases those associated with guides of rectangular, circular, sector- and ring-shaped cross-sections, with and without baffle planes.

We shall first recapitulate for ready reference the well-known relevant formulae relative to a rectangular guide. We shall then establish the corresponding formulae for a guide of sector shape leading to those for circular guides. Finally we shall proceed to the general case and identify as particular cases the results recorded for the guides of more special shape.

2. Principal Symbols

a = width of rectangle in metres

b = height of rectangle in metres

r_0 = radius of sector in metres

r_1, r_2 = inner and outer radii of ring-sector in metres

ϕ = sector angle in radians

μ, $\bar{\mu}$ = permeability of dielectric and conductor respectively in henrys per metre; for vacuum $\mu_0 = 4\pi \times 10^{-7}$

ϵ, $\bar{\epsilon}$ = permittivity of dielectric and conductor respectively in farads per metre; for vacuum, $\epsilon_0 = \frac{1}{36\pi} \times 10^{-9}$

g, $\bar{g}$ = conductivity of dielectric and conductor respectively in mhos per metre; for pure copper $\bar{g} = 5.8 \times 10^7$

$c = \frac{1}{\sqrt{\mu\epsilon}}$ = characteristic velocity of waves in the unrestricted medium in metres per second; for vacuum $c = 3 \times 10^8$

f = frequency in cycles per second

f_{0m} = cut-off frequency of the mth mode in cycles per second

$\nu_{0m} = \frac{f_{0m}}{f}$

$\omega = 2\pi f$

α_{0m} = attenuation constant corresponding to the mth mode in nepers per metre

Γ = propagation constant in the Z-direction per metre

k_{1m} = mth non-vanishing root of $J_1(x) = 0$; i.e. $k_{11} = 3.83$; $k_{12} = 7.02$; $k_{13} = 10.17$.

3. Guides of Rectangular Cross-Section

The electric and magnetic intensities of an H-wave of zero order and mth mode* in a guide of rectangular cross-section, having metal walls of infinite conductivity, and a dielectric that is nondissipative are given by

$$\left.\begin{aligned} E_x &= 0,\ E_z = 0,\ H_y = 0, \\ E_y &= -\frac{m\pi}{a} \sin\left(\frac{m\pi}{a}x\right) e^{-\Gamma z} \\ H_x &= \frac{\sqrt{1-\nu_{0m}^2}}{\sqrt{\mu/\epsilon}} \frac{m\pi}{a} \sin\left(\frac{m\pi}{a}x\right) e^{-\Gamma z} \\ H_z &= \frac{1}{j\omega\mu}\left(\frac{m\pi}{a}\right)^2 \cos\left(\frac{m\pi}{a}x\right) e^{-\Gamma z} \end{aligned}\right\} \quad (1)$$

If the conductivity of the metal walls is finite but high, the above expressions of the field become approximations but remain amply accurate for our purpose. As usual the time factor $\exp j\omega t$ is omitted.

Since both E_x and E_z are zero the lines of electric force may be represented by patterns of parallel lines stretching from top to bottom with densities that vary sinusoidally across any transverse section of the guide. For the case where $m = 1$ (Fig. 4) the electric field is a maximum along the middle line of the section and zero at each of the two side walls. This pattern is subject to the usual cyclic variation with time, that is to say, the pattern fades away, increases in the opposite direction to its original intensity,

* The characteristic feature of any H-wave is a prominent magnetic field in the direction of propagation. To rank as a wave of "zero" order it is required that in the direction of one of the transverse co-ordinates of the guide—for instance, the height of a rectangular guide—every one of the six components of the electromagnetic field is independent of that co-ordinate, that is to say, is of "zero variation" in that direction. The mode m specifies the number of half-cycles to which the non-vanishing components of the field are subjected in the direction of the other transverse co-ordinate of the guide—for instance, the width of a rectangular guide. The first mode is frequently referred to as the principal mode.

fades again, and finally re-assumes the original configuration from which the next cycle begins.

The lines of magnetic force are restricted to horizontal planes, and since both H_x and H_z are independent of the height of the guide the magnetic pattern is identical in all

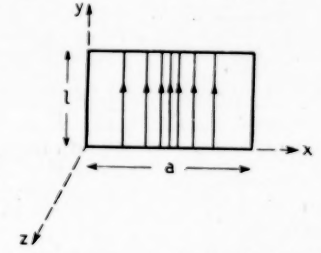

Fig. 4. *Pattern representing the electric field of an H-wave, of zero order and principal mode, propagated through a rectangular guide.*

horizontal planes. For $m = 1$ (Fig. 5) there is a single row of closed curves. The distance between recurring configurations of the pattern indicates the wavelength. With falling frequency the loops lengthen and finally, at cut-off frequency, they break up into two sets of parallel lines as shown in Fig. 6. The magnetic field is now divided into two halves pointing in *opposite* directions and being separated from one another by the vertical mid-plane (shown dotted in Fig. 6) where the magnetic intensity is

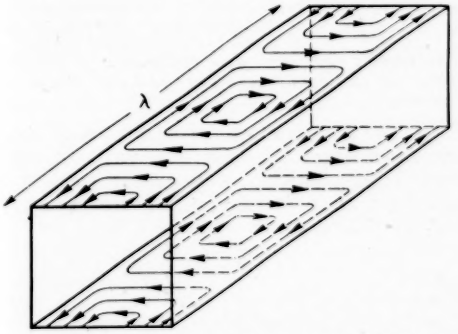

Fig. 5. *Pattern representing the magnetic field in any horizontal plane of a rectangular guide transmitting an H-wave, of zero order and principal mode. λ indicates the length of the wave.*

permanently zero. While the magnetic field is thus being broken up, no corresponding change takes place in the electric field. This field retains at cut-off frequency the character indicated in Fig. 4; that is, at any given instant the electric intensities in both halves of the guide point in the *same* direction. In consequence we are led to the conclusion that energy is simultaneously thrust from the interior towards each of the side walls whence it is reflected back towards the mid-plane.

There are thus two transverse pulsations of energy, between mid-plane and side walls, without any concurrent transmission of energy along the guide. The frequency at which this resonant state is reached is given by

$$f_{0m} = \frac{m\pi}{2\pi a} c \quad .. \quad .. \quad .. \quad .. \quad (2)$$

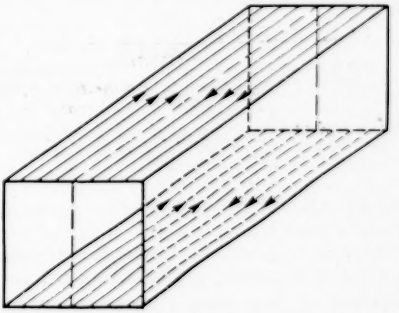

Fig. 6. *Pattern representing the magnetic field in any horizontal plane of a rectangular guide supporting an H-wave, of zero order and principal mode, at the cut-off frequency.*

Assuming that the dielectric medium is non-dissipative, as in the case of air, we have for the attenuation constant of the wave

$$\alpha_{0m} = \frac{\sqrt{\frac{\bar{\mu}\pi f}{\bar{g}}}}{\sqrt{\frac{\mu}{\epsilon}}} \left[\frac{1}{b\sqrt{1-\nu_{0m}^2}} + \frac{2}{a}\frac{\nu_{0m}^2}{\sqrt{1-\nu_{0m}^2}} \right] \quad .. \quad .. \quad (3)$$

The first term is due to currents induced in the top and bottom walls by the tangential magnetic intensities H_x and H_z. For all frequencies above $\sqrt{3}f_{0m}$ this term is an increasing function of frequency (Fig. 7). The second term arises from currents induced in the side walls. These are due to the longitudinal magnetic field H_z only; for H_x, being perpendicular to the side walls, is prevented from setting up currents in them. This second term is a decreasing function of

frequency (Fig. 7). As the cut-off frequency (2) is independent of b, the height of the guide may be increased indefinitely without

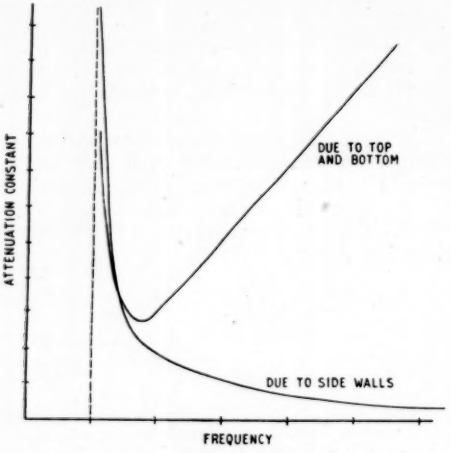

Fig. 7. *Analysis of the attenuation constant of an H-wave of zero order propagated through a guide of rectangular cross section.*

affecting the character of the wave. At the same time the first term of the attenuation constant (3) diminishes and ultimately, when the rectangular guide becomes a guide composed of two parallel planes, only the second term of (3) remains; and now, with increasing frequency the attenuation actually tends towards zero. Although this case is of no practical importance it is, as we shall presently see, the exact counterpart of a physically realizable guide, namely the guide of circular cross-section.

4. Guides of Sector Cross-Section

In so far as H-waves of zero order are concerned there is a close correlation between a guide of rectangular cross-section in a cartesian system of co-ordinates (x, y, z) and a guide of sector-shaped cross-section in a cylindrical system of co-ordinates (ρ, ϕ, z) (Fig. 8). The form of the equations of the electromagnetic field is exactly the same in the two cases; in place of the circular functions we now have Bessel functions of the first kind, and in place of the non-vanishing roots of $\sin x = 0$ we now have the non-vanishing roots of $J_1(x) = 0$, which are denoted below by k_{1m}. Thus

$$\left.\begin{aligned} &E_\rho = 0,\ E_z = 0,\ H_\phi = 0,\\ &E_\phi = -\frac{k_{1m}}{r_0} J_1\left(\frac{k_{1m}}{r_0}\rho\right) e^{-\Gamma z}\\ &H_\rho = \frac{\sqrt{1-\nu_{0m}^2}}{\sqrt{\mu/\epsilon}} \frac{k_{1m}}{r_0} J_1\left(\frac{k_{1m}}{r_0}\rho\right) e^{-\Gamma z}\\ &H_z = \frac{1}{j\omega\mu}\left(\frac{k_{1m}}{r_0}\right)^2 J_0\left(\frac{k_{1m}}{r_0}\rho\right) e^{-\Gamma z} \end{aligned}\right\} \quad (4)$$

The pattern of the electric field now consists of concentric arcs (Fig. 8) and that of the magnetic field of closed curves in radial planes (Fig. 9).

The cut-off frequency is given by

$$f_{0m} = \frac{k_{1m}}{2\pi r_0} c \qquad .. \quad .. \quad .. \quad (5)$$

which differs from the expression found for the rectangular guide merely in that the root of the Bessel function takes the place of that of the corresponding circular function.

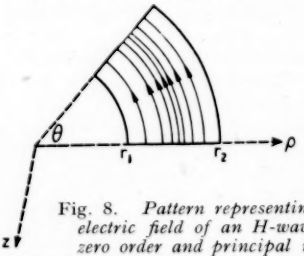

Fig. 8. *Pattern representing the electric field of an H-wave, of zero order and principal mode, propagated through a guide composed of a portion of a circular cylinder and two radial planes.*

As the attenuation constant may be calculated by Schelkunoff's well-known general formula* in a perfectly straightforward manner it may suffice here merely to state the result.

$$\alpha_{0m} = \frac{\sqrt{\frac{\mu\pi f}{\sigma}}}{\sqrt{\frac{\mu}{\epsilon}}} \left[\frac{2}{\phi r_0} C_{1m} \frac{1}{\sqrt{1-\nu_{0m}^2}} + \frac{2}{\phi r_0} \frac{\nu_{0m}^2}{\sqrt{1-\nu_{0m}^2}} + \frac{1}{r_0} \frac{\nu_{0}^2{}_m}{\sqrt{1-\nu_{0m}^2}} \right] \qquad .. \quad .. \quad .. \quad (6)$$

where

$$C_{1m} = \frac{\int_0^{k_{1m}} \left[J_1(x)\right]^2 dx}{k_{1m}\left[J_0(k_{1m})\right]^2}$$

* *Proc. I.R.E.*, 1937, Vol. 25, p. 1482.

The factor C_{1m} is always greater than unity. The lowest value occurs for $m = 1$ in which case C_{1m} is approximately 1.01. The first term in (6) arises from currents induced in the radial planes, partly by H_ρ and partly by H_z. After a minimum is passed this term is an increasing function of frequency. The second and third terms arise respectively from currents in the radial planes and in the cylindrical surface in consequence of the longitudinal field; both of these terms are decreasing functions of frequency. Since the cut-off frequency (5) is independent of the sector angle, equation (6) is applicable to the case of $\phi = 2\pi$, that is to say, to a circular guide with a single radial baffle plane (Fig. 10).

A sector angle of 2π appears to be the limiting value for a guide in physical space, but this restriction need not be applied to the equations of the field or the equations of the cut-off frequency and attenuation constant. Although originally established for physical space their range of validity extends into the region of ϕ beyond 2π—a fictitious region, known also as Riemann space,* where the sector angle may, indeed, be increased indefinitely. In consequence the length of the arc ϕr_0 increases correspondingly, with the result that the first two terms of (6) tend towards zero. As the radial planes of the guide are thus ultimately of no effect they may be removed, and now the barrier between physical and fictitious space effectively breaks down, and there remains a guide of circular cross-section. On this view then a guide of circular cross-section is the extreme case of

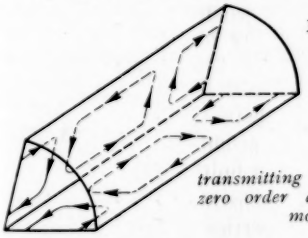

Fig. 9. *Patterns representing the magnetic field in any radial plane of a guide of sector-shaped cross-section transmitting an H-wave of zero order and principal mode.*

a guide of sector-shaped cross-section, for which the length of the arc becomes infinite—in exact correspondence to the extreme case of the rectangular guide for which the height increases indefinitely. As the first two terms in (6) vanish, the attenuation constant is supplied by the third term alone, which is a falling function with increasing frequency.

As a check of our last result we may employ a method that does not require the notion of a fictitious space. Let us assume that while the conductivity of the metal of the cylinder retains a high, but finite, value

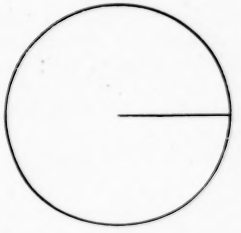

Fig. 10. *Guide composed of a circular cylinder and a radial baffle plane.*

that of the baffle plane is allowed to increase indefinitely. If then, instead of being given as a product of two factors, (6) is re-written as a sum of three separate terms, the first two terms, which refer to the baffle plane, again tend towards zero. Moreover, as the baffle plane does not constrain the field, the plane may be deleted altogether. There thus again remains a guide of circular cross-section for which the attenuation constant is as before correctly given by the third term of (6).

5. Guides of Ring-Sector Section

The treatment of this case (Fig. 11) is identical with that of the sector-shaped guide except that to the Bessel function of the first kind there must now be added that of the second kind. If we put

$$Z_n(\chi\rho) = AJ_n(\chi\rho) + BN_n(\chi\rho)$$

where A and B are real and χ is a function of the radii r_1 and r_2, the electromagnetic field is given by

$$\left.\begin{aligned}
E_\rho &= 0 \\
E_\phi &= -\frac{\nu_{1m}}{r_2 - r_1} Z_1\left(\frac{\nu_{1m}}{r_2 - r_1}\rho\right) e^{-\Gamma z} \\
E_z &= 0 \\
H_\rho &= \frac{\sqrt{1-\nu_{0m}^2}}{\sqrt{\mu/\epsilon}} \frac{\nu_{1m}}{r_2 - r_1} Z_1\left(\frac{\nu_{1m}}{r_2 - r_1}\rho\right) e^{-\Gamma z} \\
H_\phi &= 0 \\
H_z &= \frac{1}{j\omega\mu}\left(\frac{\nu_{1m}}{r_2 - r_1}\right)^2 Z_0\left(\frac{\nu_{1m}}{r_2 - r_1}\rho\right) e^{-\Gamma z}
\end{aligned}\right\} \quad (7)$$

where ν_{1m} denotes the mth root of the equation

$$J_1(\chi r_1)N_1(\chi r_2) - J_1(\chi r_2)N_1(\chi r_1) = 0;$$

moreover $m\pi \leqslant \nu_{1m} \leqslant k_{1m}$

* J. H. Jeans, "The Mathematical Theory of Electricity and Magnetism," 5th Edition, p. 283.

The cut-off frequency is given by

$$f_{0m} = \frac{\nu_{1m}}{2\pi(r_2 - r_1)}\, c \qquad \ldots \quad \ldots \quad (8)$$

If r_1 approaches zero, equations (7) and (8) become identical with those appropriate to a sector-shaped guide, that is, equations (4) and (5). If both r_1 and r_2 are very large, the Bessel functions in (7) may be replaced by their asymptotic expansions whereby (7) assumes the form of (1); i.e., that of the field within a rectangular guide.

As before, the attenuation constant may be calculated by the general formula in a straightforward manner with the result

$$\alpha_{0m} = \frac{\sqrt{\frac{\overline{\mu\pi f}}{\overline{g}}}}{\sqrt{\frac{\mu}{\epsilon}}}\left[\frac{2}{\phi}\,\frac{\int_{r_1}^{r_2}\left[Z_1(\chi\rho)\right]^2 d\rho}{r_2^2\left[Z_0(\chi r_2)\right]^2 - r_1^2\left[Z_0(\chi r_1)\right]^2}\,\frac{1}{\sqrt{1-\nu_{0m}^2}}\right.$$

$$\left. + \frac{2}{\phi}\,\frac{r_2\left[Z_0(\chi r_2)\right]^2 - r_1\left[Z_0(\chi r_1)\right]^2}{r_2^2\left[Z_0(\chi r_2)\right]^2 - r_1^2\left[Z_0(\chi r_1)\right]^2}\,\frac{\nu_{0m}^2}{\sqrt{1-\nu_{0m}^2}} + \frac{r_2\left[Z_0(\chi r_2)\right]^2 + r_1\left[Z_0(\chi r_1)\right]^2}{r_2^2\left[Z_0(\chi r_2)\right]^2 - r_1^2\left[Z_0(\chi r_1)\right]^2}\,\frac{\nu_{0m}^2}{\sqrt{1-\nu_{0m}^2}}\right] \quad (9)$$

The first term in (9) is due to the currents induced in the two radial planes, partly by H_ρ and partly by H_z. This is the rising term. The second and third terms are due to currents induced by H_z in the radial planes and in the two cylindrical surfaces respectively. If r_1 tends to zero, or if both

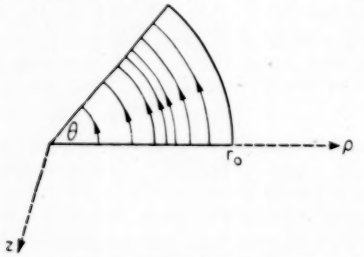

Fig. 11. *Pattern representing the electric field of an H-wave, of zero order and principal mode, propagated through a guide composed of portions of two coaxial circular cylinders and two radial planes.*

r_1 and r_2 become large, (9) assumes the forms appropriate to guides of sector or rectangular cross-sections. For $\phi = 2\pi$ the guide becomes a pair of coaxial cylinders with a single radial baffle plane (Fig. 12). As before we make use of the notion of Riemann Space and extend the validity of our equations into regions for which the sector angle exceeds 2π. And then with ϕ increasing indefinitely the first two terms of (9) again tend towards zero; and again the radial planes become ineffective and may be removed. On this view then a guide of two coaxial circular cylinders is an extreme case of a guide composed of portions of two coaxial cylinders and two radial planes and the attenuation constant is simply given by the third term in (9). This is the general case in which the attenuation approaches zero with increasing frequency.

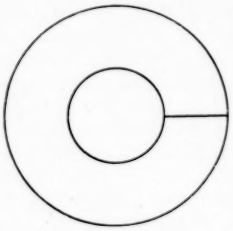

Fig. 12. *Guide composed of two coaxial circular cylinders and a radial baffle plane.*

In this, as in the special cases considered, the equations reveal that the anomalous falling off of the attenuation constant with frequency may be ascribed to the indefinite increase of one of the transverse dimensions of the guide, with the consequent disappearance of all terms that normally account for the rise with frequency. Thus all surfaces at which lines of electric force begin or end are relegated to regions infinitely far apart from one another where their existence can no longer be of any practical consequence. In the rectangular case the guide ceases thereby to be of finite height, but in the two circular cases, where the field is wrapped around an axis or around a circular cylinder, the guide retains a finite form, and the field within it, so constrained, continues to display the properties characteristic of a field of infinite range.

6. Acknowledgement

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AERIAL-TO-LINE COUPLINGS*

Cathode-Follower and Constant-Resistance Network

By R. E. Burgess, B.Sc.

(*Communication from The National Physical Laboratory*)

SUMMARY.—Considerable loss in signal/noise ratio may occur when a substantially reactive receiving aerial is connected to a receiver through a transmission line. Coupling systems which provide a match to the line at its sending end are therefore of interest. Such systems result in a reduction of the line loss which is, however, offset to a greater or lesser extent by the loss in the coupling network itself.

In the present paper two systems are considered :—(*a*) the constant-resistance network, and (*b*) the cathode follower. The overall loss in signal/noise ratio for these systems is compared with that which occurs for direct connection of the aerial to the line. The loss which occurs at the receiving end of the line is likely to be smaller and more constant with coupling networks at the sending end which match to the line than for direct connection.

Criteria are derived to show in what conditions each of the systems is preferable and these are expressed in terms of the line attenuation αl and the ratio of the aerial reactance to the characteristic impedance of the line ; i.e., $|X|/R_0$. It is found that when $|X|/R_0$ is less than about unity, direct connection is best unless the line attenuation is very large, in which case the constant-resistance network may have an advantage. If $|X|/R_0$ is greater than about 3, the cathode follower is generally best, its superiority being the greater the larger the line attenuation. The conclusions of the analysis are considered to be valid for frequencies up to about 30 Mc/s., assuming the aerial to be substantially reactive over the range concerned.

A numerical example for a typical case of a capacitive aerial operating over a wide range of frequency is given.

1. Introduction

IT is frequently required to couple a receiving aerial through a transmission line to a receiver and it is desirable that the loss in signal/noise ratio due to the line shall be a minimum over a wide frequency range.

If the aerial be substantially reactive, serious loss will usually arise from the mismatch between the impedance Z of the aerial and the characteristic impedance R_0 of the line. This loss is the greater the more pronounced the mismatch and the greater the attenuation αl of the line (α = attenuation constant, l = length). If the line were loss free ($\alpha = 0$), the signal/noise ratio would be unaffected, for the line would then behave as a purely reactive network.

It is therefore of interest to consider coupling networks between the aerial and line which provide a sending-end impedance equal to the characteristic impedance of the line over a wide band of frequencies in order that the line loss will be a minimum. The coupling network itself introduces loss and the question at issue is the extent to which the reduction in line loss is offset by the loss in the network.

Two types of network are considered in the present paper :

(i) the cathode follower, and

(ii) the constant-resistance network.

The losses due to these networks are compared with that which occurs for direct connection of the aerial, the case of a substantially reactive aerial being considered throughout.

In comparing the losses in signal/noise ratio due to the various systems, it is useful to employ the concept of " equivalent noise resistance " R_n as a measure of the loss in sensitivity introduced by the line and coupling network. This resistance R_n is defined in an analogous manner to the equivalent noise resistance of a valve :—if R is the resistive component of the aerial impedance Z, the loss in signal/noise ratio at the output terminals relative to that at the source is equal to the loss which would occur if a noise resistance R_n were connected in series with the aerial. Thus, if S_0 and N_0 are the signal and noise e.m.f.s at the aerial, and S and N are the corresponding quantities at the open-circuited output terminals of the line (Fig. 1), the relation defining R_n is

$$\left|\frac{S}{N}\right|^2 = \frac{R}{R+R_n}\left|\frac{S_0}{N_0}\right|^2 \quad .. \quad .. \quad (1a)$$

It is assumed that the effective noise temperature of the aerial is equal to the ambient temperature of the circuit elements.

The equation for R_n can be written explicitly

* MS. accepted by the Editor, February 1946.

$$R_n = R\left(\left|\frac{S_0}{S}\frac{N}{N_0}\right|^2 - 1\right) \quad .. \quad .. \quad (1b)$$

Thus, if R_n is known, the loss in signal/noise ratio can be expressed as

$$\text{Loss} = 20\log_{10}\left|\frac{S_0}{N_0}\frac{N}{S}\right|$$
$$= 10\log_{10}\left(\frac{R+R_n}{R}\right)\text{db}$$
$$= 10\log_{10}\left(\frac{R_n}{R}\right)\text{db approx.} \quad .. \quad (2)$$

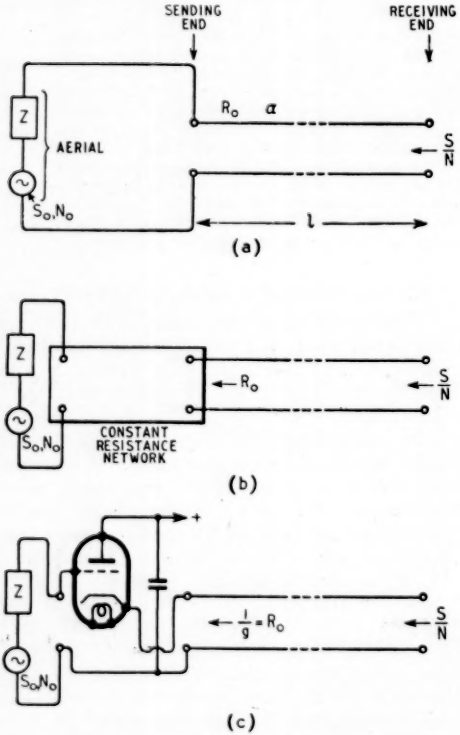

Fig. 1. *Schematic circuits of the cases considered; (a) is for direct connection of the source to the line, (b) for a constant-resistance network as a coupling, and (c) for a cathode follower.*

the latter approximation holding when R is small compared with R_n which will usually be the case for substantially reactive aerials.

It may readily be shown that the additional loss in signal/noise ratio due to connecting the receiving load impedance across the end of the line is given by

Receiving-end loss

$$= 10\log_{10}\left(1 + \frac{G_2T_2}{G_1T_1}\right)\text{db} \quad .. \quad (3)$$

where G_1 = conductance (at temperature T_1) presented by the receiving end of the line, and

G_2 = conductance of the load (at temperature T_2).

This loss must be added to the loss up to the open-circuited end of the line given by equation (2) in order to obtain the overall loss.

2. Direct Connection of the Aerial to the Line

It will be assumed throughout that the characteristic impedance of the line is a pure resistance R_0 and thus the primary constants R_1, L_1, G_1 and C_1 are related to the derived constants by the equations

$$R_0 = \sqrt{\frac{L_1}{C_1}}; \quad \alpha = \frac{R_1}{R_0} = R_0G_1 \text{ nepers/unit length (1 neper} = 8.7 \text{ db).}$$

It has been shown[1] that the loss in signal/noise ratio due to a line of attenuation αl when connected to a source of impedance $Z(= R + jX)$ is given by

$$\left|\frac{S_0}{N_0}\frac{N}{S}\right|^2 = \cosh 2\alpha l + \frac{\sinh 2\alpha l}{2R}\left(R_0 + \frac{|Z|^2}{R_0}\right)$$

and thus from equation (1b) the equivalent noise resistance is

$$R_n = R_0\sinh\alpha l\left[\frac{2R}{R_0}\sinh\alpha l + \left(1 + \frac{|Z|^2}{R_0^2}\right)\cosh\alpha l\right] \quad .. \quad (4a)$$

For a substantially reactive aerial, which is the case considered in the present paper, this equation can be simplified to

$$R_n = \tfrac{1}{2}R_0\sinh 2\alpha l\left(1 + \frac{|X|^2}{R_0^2}\right) \quad .. \quad (4b)$$

When a substantially reactive aerial is connected to a line, the impedance presented by the receiving end of the line is markedly variable with frequency. It is therefore difficult to arrange coupling of the line to the receiver which shall be suitable for operation over a wide range of frequencies[1]. This implies that the receiving end loss, given by equation (3) is likely to be appreciable at some frequencies; i.e., where the conduc-

tance G_1 presented at the receiving end of the line is small compared with the conductance G_2 of the load impedance.

A further disadvantage of direct connection lies in the existence of resonances in the combined aerial and line circuit which lead to difficulty in the maintenance of phase and amplitude equality between two or more aerials as, for example, in direction-finding systems.

These disadvantages are absent in the two matched systems discussed in the following sections since the impedance presented by the receiving end of the line is constant and resistive.

3. The Constant-Resistance Network

It is well known that certain two-terminal networks comprising resistances and reactances possess a purely resistive impedance at all frequencies[2]. In Fig. 2 three examples of this type of network are shown and in each case the impedance presented at the terminals is a constant resistance R_0 when the following condition, which is independent of frequency, is satisfied :

$$L/C = R_0^2$$

In the present problem L or C would represent the substantially reactive aerial and the two terminals would be connected to the line. It is assumed that the resistance of the aerial and of the other reactive elements is negligible and that the effective L or C of the aerial is substantially independent of frequency over the range of operating frequencies.

In all these circuits the voltage transfer ratio S/S_0 to the end of the line is given by

$$\left|\frac{S_0}{S}\right|^2 = \left(1 + \frac{|X|^2}{R_0^2}\right) e^{2al}$$

where X is the aerial reactance at the frequency considered. Thus the equivalent noise resistance is given by

$$R_n = R_0\left(1 + \frac{|X|^2}{R_0^2}\right) e^{2al} \quad .. \quad .. \quad (5)$$

The additional loss when the line is terminated in its characteristic impedance at the receiving end is 3 db.

4. The Cathode Follower

It is assumed that the input admittance of the valve is negligible compared with the admittance of the aerial. Then it may be shown[3] that the equivalent noise resistance of a triode cathode-follower stage having an infinite cathode load is given by

$$R_n = R_v = n/g$$

where R_v = equivalent noise resistance of the valve.

g = mutual conductance of the valve

$n \equiv gR_v$ = a numerical factor depending upon the valve structure and cathode temperature.

Since the output impedance of the cathode follower is $1/g$ and the transfer ratio of signal voltage between input and output is unity, the factor n is readily identified as the ratio of the equivalent noise temperature of the output impedance $1/g$ to the ambient temperature T. Theory and experiment[4] show that n has a value of approximately 2.5 for triodes with oxide-coated cathodes at about 1000° K, which is the condition in most modern valves.

If a line of characteristic impedance

$$R_0 = 1/g$$

be attached to the cathode follower (Fig. 1c) it will be matched at its sending end and the signal at the receiving end is given by

$$S = S_0\, e^{-al}$$

Although the impedance at the end of the line is still R_0, the noise voltage there will be

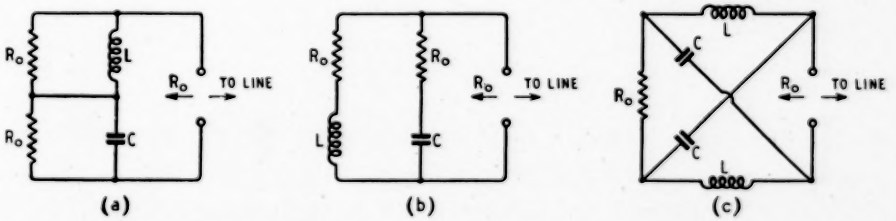

Fig. 2. *Constant-resistance networks suitable for coupling a reactive source (L or C) to a line of characteristic impedance R_0. The circuits are :—(a) full-series type, (b) full-shunt type and (c) lattice type. In all cases the constant resistance is $L/C = R_0^2$.*

less than at the cathode, since the noise from the valve (at effective temperature nT) is attenuated and replaced by noise from the line at the lower temperature T. It can be shown that the square of the noise voltage at the receiving end of the line is less than that at the sending end by the factor

$$\{1 + (n - 1)\, e^{-2\alpha l}\}/n$$

The overall effect of the line is to increase the square of the noise/signal ratio by the factor

$$(e^{2\alpha l} + n - 1)/n$$

Thus, the equivalent noise resistance of the cathode follower with the line is given by

$$R_n = R_T \frac{e^{2\alpha l} + n - 1}{n}$$

$$= R_0\,(e^{2\alpha l} + 1.5) \text{ when } n = 2.5 \quad (6)$$

Since the impedance presented at the receiving end of the line is effectively at a temperature T_1 (above the ambient temperature T the loss in signal/noise ratio due to terminating the line in its characteristic impedance will be less than 3 db.

From the above, the temperature T_1 is found to have the value

$$T_1 = T\,(1 + 1.5\, e^{-2\alpha l})$$

and thus in the matched condition

$$G_1 = G_2 = \frac{1}{R_0}\,; \quad T_2 = T$$

which gives from equation (3) for the receiving-end loss

$$10 \log_{10}\left(\frac{2 + 1.5\, e^{-2\alpha l}}{1 + 1.5\, e^{-2\alpha l}}\right) \text{ db}$$

which lies between 1.5 db for αl small and 3 db for αl large.

5. Comparison of the Methods of Coupling

The following conclusions may be drawn from the results derived in the preceding sections :—

(a) In the case of direct connection, the impedance appearing at the receiving end of the line is variable with frequency and considerable additional loss [cf. equation (3)] may occur at some frequencies. In the case of the constant-resistance network and the cathode follower, the receiving-end loss can be made small since the impedance presented at the end of the line is a constant resistance. It is therefore necessary to bear in mind the likelihood of the greater receiving-end loss of direct connection when comparing it with the matched systems.

(b) The cathode follower is superior to direct connection when the magnitude of the aerial reactance is greater than the value given by equating the right-hand sides of equations (4b) and (6) :

$$\frac{|X|}{R_0} = \sqrt{\frac{2\, e^{2\alpha l} + 3}{\sinh 2\alpha l} - 1} \quad .. \quad .. \quad (7)$$

This function of αl has been plotted in Fig. 3 which enables the useful range of application of the cathode follower to be estimated. As αl increases, it tends to a limiting value of 1.73, while for small values of αl it increases indefinitely.

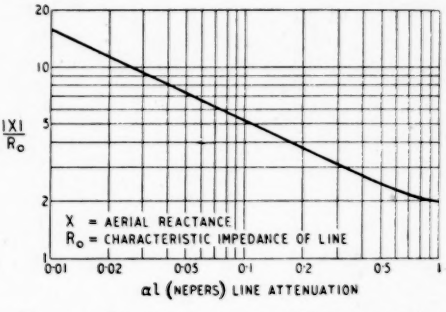

Fig. 3. *The magnitude of aerial reactance for which the cathode follower and direct connection give the same signal/noise ratio at the end of the line.*

By virtue of the considerations in (a) regarding the receiving-end loss, the cathode follower will in practice be advantageous with aerials of reactance somewhat lower than indicated by Fig. 3. Curves in Fig. 4 show the gain in signal/noise ratio provided at the end of the line by the cathode follower relative to direct connection. The intersections with the zero line occur at the values of $|X|/R_0$ already given in Fig. 3. The upper limit to the gain is represented by the curve for $\alpha l = \infty$.

(c) The constant-resistance network is inferior to direct connection as may be seen from comparing equations (4b) and (5) which give for the ratio of the equivalent noise resistances :

$$\frac{R_n \text{ (const. res. network)}}{R_n \text{ (direct connection)}} = \frac{2e^{2\alpha l}}{\sinh 2\alpha l} = \frac{4}{1 - e^{-4\alpha l}} \quad .. \quad (8)$$

The corresponding loss in signal/noise ratio of the constant-resistance network relative to the direct connection is shown in Fig. 5 from which it is seen that this loss has a

lower limit of 6 db for αl large. The constant-resistance network is likely to be of practical value only when the line attenuation αl is large and the aerial reactance is small. In this condition it is superior to the cathode follower if

$$|X|/R_0 < 1.22e^{-\alpha l}$$

but it will always be inferior to the direct connection. When αl is large, this inferiority is a minimum and may be more than offset by the constant resistance presented at the receiving end, as indicated in (*a*).

(d) The conclusions of the analysis are considered to be valid for frequencies up to the order of 30 Mc/s provided that the main hypothesis of a substantially reactive aerial is still satisfied. At higher frequencies the input admittance and inter-electrode capacitances of the cathode follower are no longer negligible. In practice, however, at V.H.F. only direct connection of the aerial to the line is likely to be suitable, since the aerial will usually be of relatively low reactance if not actually resonant.

The only significance of bandwidth (not to be confused with the range of operating frequency) is that the impedances concerned shall be substantially constant within it.

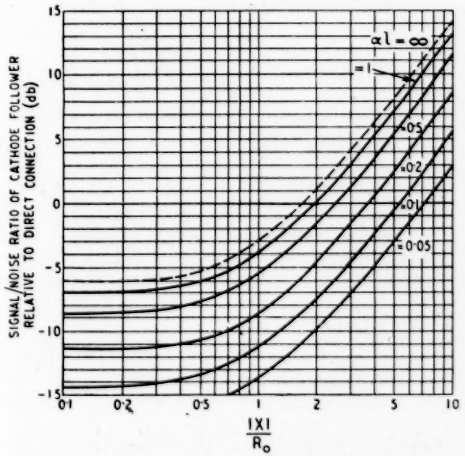

Fig. 4. *Comparison of the signal/noise ratio of the cathode follower and direct connection for various values of line attenuation αl (nepers).*

6. Numerical Example

It may be of interest to consider a typical numerical example. An aerial has a capacitance of 50 $\mu\mu$F and is required to work into a line of characteristic impedance 100 ohms and attenuation 0.2 neper (1.7 db) over the frequency band 3 to 10 Mc/s.

The aerial reactance $|X|$ varies between 1000 ohms and 300 ohms and thus $|X|/R_0$ varies between 10 and 3. From Fig. 4 the signal/noise ratio for the cathode follower is seen to be between + 8.5 db and − 1.5 db relative to direct connection ; in view of the matching property of the cathode follower, it may be regarded as markedly superior to

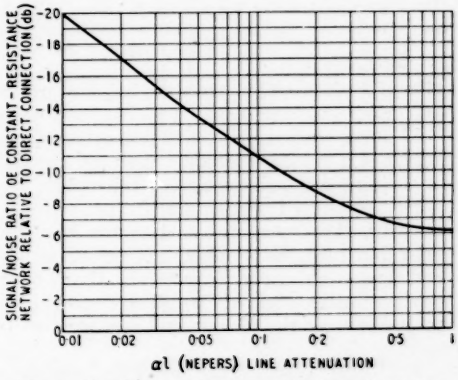

Fig. 5. *Comparison of the signal/noise ratio of the constant-resistance network and direct connection.*

direct connection. From Fig. 5 it is seen that the constant-resistance network provides a signal/noise ratio 8.5 db down on the direct connection and is therefore much inferior to the cathode follower.

If the line is terminated in its characteristic impedance at the receiving end the additional loss is 1.8 db for the cathode follower and would be 3 db for the constant-resistance network. In the case of direct connection the receiving end loss is dependent on the electrical length of the line and is likely to vary between wide limits.

7. Acknowledgements

The work described above was conducted as part of the programme of the Radio Research Board to whom this paper was first circulated in 1943. It is now published by permission of the Department of Scientific and Industrial Research.

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OSCILLATION HYSTERESIS IN GRID DETECTORS*

By E. E. Zepler

SUMMARY.—The conditions under which grid-leak detectors employing variable regeneration may exhibit oscillation hysteresis are discussed. A theory covering the principal effects responsible for hysteresis is given, together with its experimental verification. Various measures against oscillation hysteresis are recommended.

1. Introduction

REGENERATIVE grid-leak detectors are now less widely used than they were some years ago. Nevertheless, in the hands of a skilled operator, they are capable of giving more gain and selectivity than any other circuit of equal simplicity. Consequently, the regenerative detector has considerable usefulness when saving of weight and space is more important than tonal quality and ease of handling, and it is probable that the circuit will continue to be used in these cases.

For the circuit to produce optimum results it is important that the regeneration should work smoothly; i.e., the amplitude of oscillation should increase with increasing regeneration steadily and without discontinuities, and decrease in the same manner when the regeneration is decreased. In the case of jumpy regeneration, known as oscillation hysteresis or backlash, oscillation starts suddenly with a finite, sometimes fairly large, amplitude. If the regeneration is then decreased the amplitude falls steadily to some finite value, after which oscillation stops suddenly for a value of regeneration well below that for which it started. In this case a valve adjusted near the point of oscillation may oscillate strongly when a signal is applied and maintain oscillation after the signal has been removed. It is evident that under these conditions the valve cannot be used with maximum efficiency.

Oscillation hysteresis was a serious problem in the early development of valve oscillators some 25 years ago. The effect was due to the valves being worked with a fairly large fixed grid bias. Under these conditions it is possible for the average mutual conductance of a valve to increase at first with increasing amplitude, until eventually it falls again (Fig. 1). If, therefore, the regeneration is adjusted so that oscillation can just begin, the state of oscillation is not stable, because with increasing amplitude the feedback factor $A\beta$ increases. The effect has been discussed and has been rightly attributed to the peculiarities of the valve characteristic.† With the introduction of the grid-leak resistance and capacitance, oscillation hysteresis in ordinary oscillators has been largely overcome because of the self-biasing effect of the valve. The grid bias produced by the grid current is not constant, but increases with increasing amplitude of oscillation; this causes the mutual conductance to drop and thus prevents hysteresis.

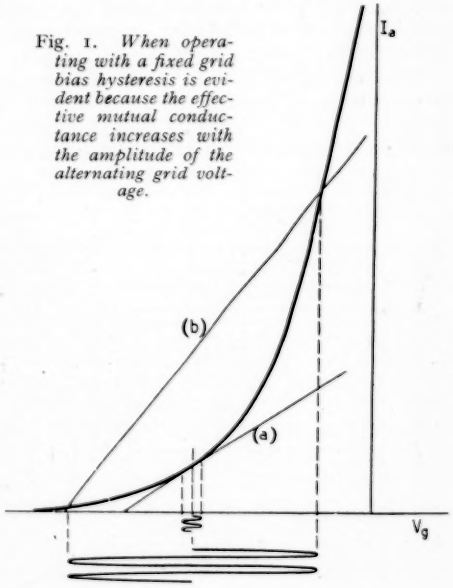


Fig. 1. *When operating with a fixed grid bias hysteresis is evident because the effective mutual conductance increases with the amplitude of the alternating grid voltage.*

Even with this self-biasing effect hysteresis is frequently met with in grid leak detectors.

* MS. accepted by the Editor, February 1946.

† E. V. Appleton and B. Van Der Pol, "On a Type of Oscillation Hysteresis in a simple Triode Generator," *Phil. Mag.*, January 1922.

It is almost absent on short waves, quite marked on medium waves and very strong towards the longer wavelengths. This fact suggests that there are additional causes other than the valve characteristic. When examining these causes we consider as the basic fact that oscillation hysteresis is present whenever the feedback factor $A\beta$ increases with increasing amplitude. But, in calculating the feedback factor as a function of amplitude, we have to keep in mind that the frequency of oscillation may be a function of amplitude; i.e., the feedback factor must be calculated for that frequency at which $A\beta$ is real. Hence, if V_g is the alternating grid voltage and f the frequency of oscillation, the condition for the presence of hysteresis is that

$$\frac{d(A\beta)}{dV_g} > 0, \text{ where } \frac{d(A\beta)}{dV_g} = \frac{\partial(A\beta)}{\partial V_g}(f=\text{const}) + \frac{\partial(A\beta)}{\partial f}(V_g=\text{const}) \times \frac{df}{dV_g}.$$

Among the various factors that may produce oscillation hysteresis in a grid-leak detector the most important are (*a*) variation in the Q-factor of the tuned grid circuit due to grid current, (*b*) phase displacement of the anode current because of the reactance of the feedback capacitor. The two causes will be discussed separately.

2. Q-Factor Variation

The basic circuit in Fig. 2(*a*) may be treated as an ordinary feedback amplifier. For the sake of simplicity we assume that the reactance of the anode coil and the transferred impedance of the grid circuit are small compared with the valve impedance; then the anode current is always in phase with the grid voltage. From the equivalent circuit shown in Fig. 2 (*b*) we obtain

$$I_a = g_m V_g, \text{ and}$$

$$V_g' = I_a j\omega M \frac{1/j\omega C_g}{r + j(\omega L - 1/\omega C_g)} = V_g g_m \frac{M/C_g}{r + j(\omega L - 1/\omega C_g)}$$

The phase condition is fulfilled when $\omega L = 1/\omega C_g$, i.e. for the resonant frequency $\omega_0/2\pi$ of the grid circuit, independently of the amplitude of oscillation (df/dV_g in the above equation is zero). Then the feedback factor becomes $\frac{V_g'}{V_g} = \frac{g_m M}{r C_g} = g_m \omega_o M Q$, where Q is the magnification factor of the grid circuit. Oscillation starts when $M = \frac{1}{g_m \omega_o Q}$.

The amplitude of oscillation should then be infinitely small and should be prevented from increasing by the decrease of g_m. In actual fact, however, the grid-cathode path of the valve is equivalent to a resistance which increases with increasing amplitude of oscil-

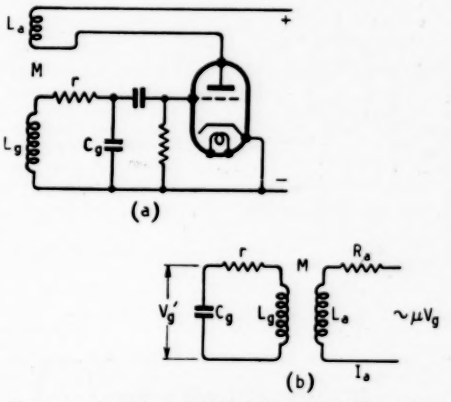


Fig. 2. *A typical grid-leak biased oscillator is shown at (a) with its equivalent circuit at (b).*

lation. Therefore, in the expression for the feedback factor, both g_m and Q are functions of amplitude. If, with increasing amplitude, the fractional increase of Q is larger than the fractional decrease of g_m, the differential coefficient $\frac{d(A\beta)}{dV_g} = \omega_0 M \frac{d(g_m Q)}{dV_g}$ is positive and oscillation hysteresis is present.

3. Experimental

As a check on the above theory both the mutual conductance and the equivalent grid-cathode resistance of the oscillator valve were measured as a function of the alternating voltage between grid and cathode. The methods used are shown in Fig. 3 (*a* and *b*), the circuits being of a conventional bridge type. In these measurements the valve acted in a way typical of its behaviour as an oscillator valve; i.e., when the voltage V_g across AB increased, the working bias of the valve increased as well, so that g_m decreased while the equivalent resistance between A and B increased. The measuring frequency was identical with the frequency at which the valve was used as an oscillator, the indicator being tuned to the measuring frequency to rule out the influence of harmonics caused by the valve. (Measurements at audio frequency with a telephone as indicator gave

identical results, provided the grid leak capacitor was increased inversely proportional to the frequency).

The results are shown in Fig. 4. The value of R_g includes the grid-leak resistance ($1M\Omega$), in accordance with the practical conditions; the grid capacitor was 100 $\mu\mu$F. The valve used was a type ML4 working with 100 volts H.T. In the tuned grid circuit shown in Fig. 2 (*a*) $C_g = 500\ \mu\mu F$, $f_0 = 230$ kc/s, $Q_0 = 90$, where Q_0 refers to the circuit alone. For the calculation of the feedback factor as a function of amplitude a knowledge of the Q-factor under working conditions is required. This is found from Q_0 and from R_g by the relation $1/Q = 1/Q_0 + \omega_0 L/R_g$. The values of Q and of $g_m Q$ are plotted in Fig. 5, as a function of the voltage between A and B.

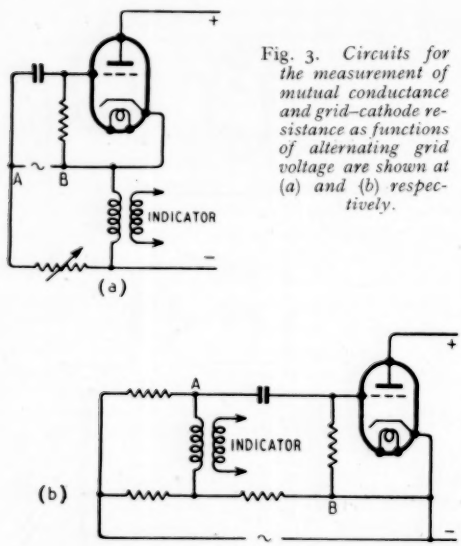


Fig. 3. *Circuits for the measurement of mutual conductance and grid–cathode resistance as functions of alternating grid voltage are shown at (a) and (b) respectively.*

We see that, although g_m decreases with increasing V_g, the feedback factor increases at first and then falls off. Therefore it follows that, as soon as M has reached the value necessary for oscillation to start, the amplitude of oscillation will increase until the factor $g_m Q$ has again reached the initial value of 200. Oscillation hysteresis is fairly prominent, oscillation starting with a grid amplitude of 3.7 volts. If now M is decreased, stable oscillation can persist, until the value of $g_m Q$ becomes approximately 262. The grid amplitude is then only 1 volt and oscillation must break off suddenly if M is further decreased.

The condition under which hysteresis would just be absent is that

$$\frac{d(A\beta)}{dV_g} = 0 \text{ for } V_g = 0$$

that is

$$\omega_0 M\left(\frac{dg_m}{dV_g}Q + \frac{dQ}{dV_g}g_m\right) = 0,$$

and therefore

$$\frac{dg_m}{dV_g}\frac{1}{g_m} = -\frac{dQ}{dV_g}\frac{1}{Q}$$

If Z_0 is the impedance of the parallel-tuned circuit without additional grid damping, then $Q_0 = Z_0/\omega_0 L$, and the Q-factor under working conditions is $Q = \dfrac{Z_0 R_g}{Z_0 + R_g}\dfrac{1}{\omega_0 L}$, hence

$$\frac{dQ}{dV_g} = \frac{dQ}{dR_g}\frac{dR_g}{dV_g} = \frac{{Z_0}^2}{(Z_0 + R_g)^2\,\omega_0 L}\frac{dR_g}{dV_g}$$

$$\frac{1}{Q}\frac{dQ}{dV_g} = \frac{1}{R_g}\frac{dR_g}{dV_g}\frac{Z_0}{Z_0 + R_g}$$

From Fig. 4 we see that the fractional change of g_m for zero grid amplitude is approximately

$$\frac{1}{g_m}\frac{dg_m}{dV_g}_{(V_g = 0)} = -0.05 \text{ per volt } V_g;$$

the fractional change of R_g is 4.5 per volt V_g. Hence oscillation hysteresis will be absent when $4.5\dfrac{Z_0}{Z_0 + R_g} < 0.05$; i.e., when $Z_0 < 2{,}900$ ohms. This explains why in the above example a resistance of 10,000 ohms placed in parallel with the grid circuit did not prevent hysteresis, but only reduced it so that the initial amplitude of oscillation was about 0.5 volt. The absence of hysteresis on short waves is due to the fact that the impedances of parallel tuned circuits are

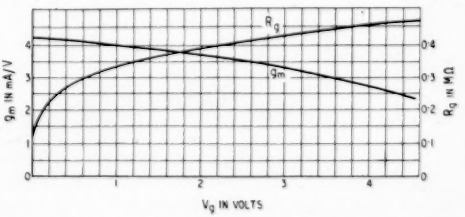


Fig. 4. *Mutual conductance and grid–cathode resistance of a ML4 valve with 100 volts H.T. supply.*

usually a few thousand ohms, whereas R_g is independent of frequency (unless transit time effects are considered).

Conditions may be improved in various ways such as lowering the H.T. or increasing the grid-leak resistance. The former moves the working point of the valve nearer to the lower bend of the characteristic where g_m decreases more quickly with increasing bias. The second measure increases the bias for

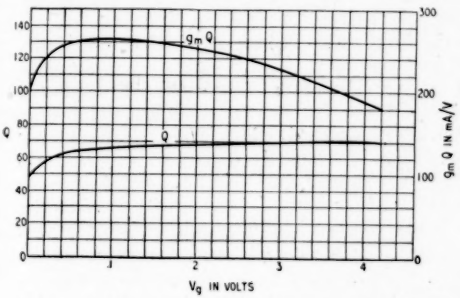


Fig. 5. *Values of effective Q and g_mQ for the valve of Fig. 4 with a tuned circuit having $Q = 90$ at 230 kc/s.*

zero grid amplitude so that R_g increases. When, in the above example, an H.T. of 75 volts and a grid leak resistance of 8 megohms were used, oscillation started with an amplitude of only 0.3 volt, instead of about 4 volts with 1 MΩ and 100 volts H.T.

If the anode load is a large resistance, the voltage at the anode rises with increasing bias. This keeps g_m more constant with increasing bias and tends to increase hysteresis. A diode, biased to its cut-off point and placed in parallel with the tuned circuit, easily eliminates hysteresis in this case, due to the Q-factor decreasing with increasing amplitude. It is also effective when the hysteresis is caused by the anode current having a phase displacement with respect to the grid voltage.

4. Anode Current Phase Angle

The conventional circuit of a grid-leak detector using a variable feedback capacitor may be seen in Fig. 6. The A.F. load in the anode circuit has been omitted in order to simplify the conditions. If we neglect, as in Sect. 2, the reactance of L_a and the transferred impedance of the grid circuit, the feedback factor of the circuit becomes:

$$A\beta = \frac{V'_g}{V_g} = \mu \frac{M}{C_g} \frac{1}{\left(R_a - \frac{j}{\omega C_a}\right)\left(r + j\omega L_g - \frac{j}{\omega C_g}\right)}$$

The phase is correct for oscillation when the imaginary terms disappear, hence:

$$R_a\left(\omega L_g - \frac{1}{\omega C_g}\right) - \frac{r}{\omega C_a} = 0$$

$$\omega^2 = \frac{1}{L_g C_g} + \frac{r}{R_a L_g C_a}$$

$$\omega \approx \omega_0\left(1 + \frac{1}{2}\frac{rC_g}{R_a C_a}\right)$$

From this it is seen that oscillation is possible only at a frequency higher than the resonant frequency of the grid circuit. The deviation from resonance must be such that the phase displacement of the anode current caused by the reactance of C_a is corrected in the grid circuit. The influence of C_a advances the phase hence the R.F. current in the grid circuit must lag behind the e.m.f. induced in L_g. This is the case if the frequency of oscillation is higher than the resonant frequency of the grid circuit. Under these conditions the feedback factor is:

$$\frac{V_g'}{V_g} = \mu \frac{M}{C_g} \cdot \frac{1}{rR_a + \frac{L_g}{C_a} - \frac{1}{\omega^2 C_a C_g}}$$

where ω is a function of R_a and is given by the above relation. Hence:

$$\frac{V_g'}{V_g} = \mu\frac{M}{C_g}\frac{1}{rR_a + \frac{L_g}{C_a} - \frac{1}{C_a C_g\left(\frac{1}{L_g C_g} + \frac{r}{R_a L_g C_a}\right)}} = \mu\frac{M}{C_g}\frac{1}{rR_a + \frac{L_g}{C_a}\frac{rC_g}{rC_g + R_a C_a}}$$

The feedback factor, expressed as a function of R_a, becomes a maximum when

$$r - \frac{L_g}{C_a}\frac{rC_g C_a}{(rC_g + R_a C_a)^2} = 0$$

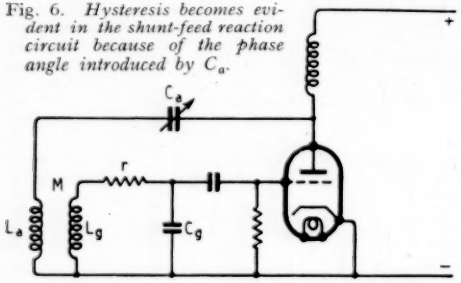
$R_a C_a$ is large compared with rC_g, whence

$$R_a^2 C_a^2 = L_g C_g = \frac{1}{\omega_0^2}$$

$$\therefore\ R_a = \frac{1}{\omega_0 C_a}$$

From this it follows that hysteresis exists when, for infinitely small grid amplitudes, the valve impedance is less than $1/\omega_0 C_a$. With increasing amplitude R_a increases, the frequency for which $A\beta$ is real decreases and $A\beta$ itself goes up. If, under these conditions, C_a or M has been adjusted to a value such that oscillation can just start, i.e. such that $A\beta = 1$, the amplitude will increase until R_a reaches the second value for

Fig. 6. *Hysteresis becomes evident in the shunt-feed reaction circuit because of the phase angle introduced by C_a.*



which the feedback factor is unity. The phase difference between anode current and grid voltage is then less than 45°, and the oscillation is stable (squegging is not taken into consideration). If now M or C_a is decreased, the amplitude of oscillation falls steadily until R_a equals the reactance of C_a, after which oscillation ceases suddenly.

The two values of R_a for which the feedback factor is unity may be found from the above equation, viz.

$$1 = \mu \frac{M}{C_g} \frac{1}{rR_a + \frac{L_g}{C_a} \cdot \frac{rC_g}{rC_g + R_a C_a}}$$

rC_g can be neglected in comparison with $R_a C_a$, hence

$$R_a = \frac{\mu M}{2rC_g} \pm \sqrt{\left(\frac{\mu M}{2rC_g}\right)^2 - \frac{L_g C_g}{C_a^2}}$$

$$= \frac{\mu\omega_0 MQ}{2} \pm \sqrt{\left(\frac{\mu\omega_0 MQ}{2}\right)^2 - \left(\frac{1}{\omega_0 C_a}\right)^2}$$

We assume that M is fixed and that the value of R_a for infinitely small amplitudes is known, then the magnitude of C_a for which oscillation starts is obtained from the last equation. The same equation then gives the second value of R_a for which the feedback factor is unity.

The above conclusions were checked experimentally, and results agreed well with the theory. In order to eliminate the effect of grid damping, the circuit shown in Fig. 7 was used. The self-biasing effect is maintained, but the oscillator valve never passes grid current. A circuit of this type may prove useful in practice.

The result of another experiment is of theoretical interest. The oscillator valve was operated with a fixed bias so that the working point was near the anode bend. Then hysteresis was present when C_a was very large; this was explained at the beginning as being due to an increase of g_m with amplitude. Since μ is fairly constant, an increase of g_m is equivalent to a decrease of R_a. Therefore hysteresis disappeared when M was increased to such a value that oscillation started when $1/\omega_0 C_a$ was larger than R_a. Numerical agreement was not too good, no doubt due to the fact that with small anode currents μ varies with amplitude.

When the oscillator triode of Fig. 7 was replaced by a pentode and the R.F. choke by a resistance of 5,000 ohms, there was no sign of hysteresis for any value of C_a. This

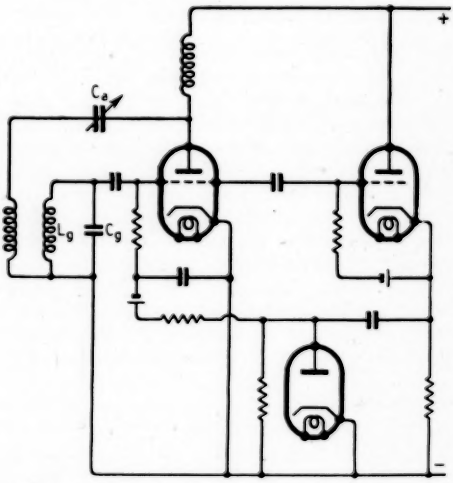


Fig. 7. *Circuit used to check the effect of C_a on hysteresis. A diode is used to obtain grid bias so as to avoid grid current effects.*

is to be expected because the phase displacement of the R.F. current flowing through C_a is determined by the reactance of C_a, and by the 5,000 ohm resistance. The valve impedance is so large that its influence on the phase is negligible. There remains only the decrease of g_m with increasing amplitude which leads to smooth starting of oscillation.

Experience shows that, even when hysteresis is absent, the amount of regeneration which can be applied when receiving a modulated carrier is often less than might be expected. The larger the signal the less satisfactory is the effect of regeneration. This is easily understood from the above. Usually, when hysteresis is absent, the feedback factor falls quickly with increasing signal strength. When the signal is modulated, the regeneration must be adjusted so that oscillation is absent when the signal amplitude is zero. The amount of effective regeneration is therefore smaller for a larger carrier. For this reason high selectivity with a large signal can only be obtained if the graph of $A\beta$ against amplitude is almost horizontal for low amplitudes, and if the frequency for which $A\beta$ is real is independent of amplitude. With a well designed circuit a bandwidth of a few hundred cycles per second can be achieved at frequencies as high as 10 Mc/s.

MODULATION PRODUCTS*

Calculation from Equidistant Ordinates

By A. Bloch, Dr.-Ing., M.Sc.

(Communication from the Staff of the Research Laboratories of The General Electric Company, Limited, Wembley, England.)

SUMMARY.—Tables and formulae for the calculation of modulation products from valve characteristics are given which are similar to those customarily used for the determination of the amplitude of harmonics from a series of equidistant ordinates. The derivation of the method is also described.

Introduction

IF a valve possesses a non-linear characteristic and the input e.m.f. contains more than one frequency, the output contains not only these components and their plain harmonics but also all possible additive and subtractive combinations thereof, the so-called intermodulation products.† The purpose of the present article is to put on record a set of formulae for the calculation of their amplitudes from equidistant ordinates of the valve characteristic; they are quite similar to the well-known formulae for the calculation of the harmonic output in the case of one input frequency.‡

Two tables are given with formulae ready for use, which cover the cases most likely to be encountered; these are followed by the derivation of the formulae. It will be seen that the new formulae follow very easily from those used for the calculation of harmonic output for a single input frequency, so that there is no necessity for extensive tabulation. It will also be seen that it is just as easy to construct formulae which deal with more than two input frequencies.

Description of Tables

The Tables give the coefficients with which the equidistant ordinates have to be multiplied, these ordinates being numbered so that the one coinciding with the working point of the valve has the number zero. A factor common to all the coefficients in one line appears in the column headed "Common Factor."

The amplitudes of the input e.m.fs. of different frequencies are denoted by "A" and "B," these amplitudes being measured in units which equal the potential difference between two neighbouring ordinates.

We read, for instance, from Table I that

* MS. accepted by the Editor, February 1946.

† This term includes, of course, "fundamentals" and "simple harmonics"; such components of the output correspond to modulation products in which the "other" frequencies participate to zero order (e.g., the output component $2\omega_a$ arises as modulation product $2\omega_a + 0.\omega_b + \ldots$)

‡ Sets of such formulae, giving up to the 6th harmonic and taking into account up to 7 equally spaced ordinates of the characteristic, were given by D. C. Espley in *Proc. I.R.E.*, Vol. 21 (1933), pp. 1439–1446.

TABLE I: COEFFICIENTS $r_k/2$ FOR OUTPUT $\omega_a \pm \omega_b$

Amplitudes A	Amplitudes B	Common Factor	Ordinates −4	−3	−2	−1	0	+1	+2	+3	+4
1	1	1/8			+ 1	0	− 2	0	+ 1		
1	2	1/12		+ 1	+ 1	− 1	− 2	− 1	+ 1	+ 1	
1	3	1/2560	+ 167	+ 252	− 212	− 252	+ 90	− 252	− 212	+ 252	+ 167
2	2	1/18	+ 1	+ 2	+ 1	− 2	− 4	− 2	+ 1	+ 2	+ 1

TABLE II: COEFFICIENTS $r_k/2$ FOR OUTPUT $\omega_a \pm 2\omega_b$

Amplitudes A	Amplitudes B	Common Factor	Ordinates −4	−3	−2	−1	0	+1	+2	+3	+4
1	1	1/16			− 1	+ 2	0	− 2	+ 1		
1	2	1/16		− 1	0	+ 3	0	− 3	0	+ 1	
1	3	1/10240	− 559	− 486	+ 1774	+ 146	0	− 146	− 1774	+ 486	+ 559
2	1	1/24		− 1	+ 1	+ 1	0	− 1	− 1	+ 1	
2	2	1/24	− 1	− 1	+ 2	+ 3	0	− 3	− 2	+ 1	+ 1
3	1	1/5120	− 167	+ 82	+ 382	− 342	0	+ 342	− 382	− 82	+ 167

the output of frequency $\omega_a + \omega_b$, for the case where the input frequencies ω_a and ω_b have each an amplitude of unity, is given by

$$\tfrac{1}{8}[I_{-2} - 2I_0 + I_{+2}]$$

Derivation of Formulae

If an excitation $e_g = A\cos\omega_a t$ is applied to the grid of the valve we get an output of the form

$$i_a = C_0 + C_1\cos\omega_a t + C_2\cos 2\omega_a t + \ldots = \sum_{p=0}^{p=P} C_p\cos p\omega_a t \quad \ldots \quad \ldots \quad (1)$$

Each of the coefficients C_p can be calculated by an expression of the type

$$C_p = \sum_{m=-M}^{m=+M} a_{pm} \cdot I_m \quad \ldots \quad \ldots \quad (2)$$

where the values of I_m are the current values for equidistant steps of grid potential given by the valve characteristic, and those of a_{pm} are known constants (given, for instance, in Espley's paper cited above).

In the case of a non-linear characteristic the values C_p will depend on the working potential of the valve. If a sufficient number of current ordinates are available we are then enabled to calculate a whole series of values of C_p by assuming the working potential to be shifted so as to coincide with one or the other of the given ordinates of the characteristic. The values of C_p obtained in this way are then most suitably labelled $C_p(-n) \ldots C_p(-1)$, $C_p(0) \ldots C_p(+n)$ the argument in brackets denoting the working potential of the valve to which they refer.

Now, quite generally, if the working potential of the valve is varied in a sinusoidal fashion, say, by an amount $B\cos\omega_b t$, then C_p will itself become a function of time of the form

$$C_p = C_{p0} + C_{p1}\cos\omega_b t + \ldots = \sum_{q=0}^{q=Q} C_{pq}\cos q\omega_b t \quad \ldots \quad \ldots \quad (3)$$

The values of C_{pq} can obviously be calculated from equidistant values of C_p by a formula of the same type as (2), i.e.

$$C_{pq} = \sum_{n=-N}^{n=+N} b_{qn} \cdot C_p(n) \quad \ldots \quad \ldots \quad (4)$$

From (3) it follows that the valve output has the form

$$i_a = \sum_p C_p\cos p\omega_a t = \sum_p \Big(\sum_q C_{pq}\cos q\omega_b t\Big) \cdot \cos p\omega_a t$$

$$= \sum_p \sum_q C_{pq} \cos p\omega_a t \,.\, \cos q\omega_b t$$

$$= \tfrac{1}{2} \sum_p \sum_q C_{pq}(p\omega_a + q\omega_b)t \qquad \ldots \quad \ldots \quad (5)$$

so that the coefficients C_{pq} equal just twice the wanted intermodulation products.

From (2) it follows that

$$C_p(n) = \sum_{m=-M}^{m=+M} a_{pm} \,.\, I_{m+n} \qquad \ldots \quad \ldots \quad (2a)$$

hence from (4)

$$C_{pq} = \sum_{n=-N}^{n=+N} b_{qn} \,.\, \sum_{m=-M}^{m=+M} a_{pm} \,.\, I_{m+n} \qquad \ldots \quad (6)$$

$$r_{-1} = \tfrac{1}{2} \times \tfrac{1}{3} \times (-1 + 0 + 0) = -\tfrac{1}{6}$$

The "mechanism" by means of which the coefficients r_k are created becomes perhaps clearer still if we follow the course of the calculation in matrix notation. It will be noted that formula (2) can be interpreted as the multiplication of a "line" vector $|a_{pm}|$ by a "column" vector $\{I_m\}$. In the case just mentioned ($A = 2$, $p = 1$) the line vector has the form

$$|\,-1 \quad -1 \quad 0 \quad +1 \quad +1\,|$$

It is then easily seen that

$$\begin{vmatrix} C_1(-1) \\ C_1(0) \\ C_1(+1) \end{vmatrix} = \tfrac{1}{3} \begin{vmatrix} -1 & -1 & 0 & +1 & +1 & 0 & 0 \\ 0 & -1 & -1 & 0 & +1 & +1 & 0 \\ 0 & 0 & -1 & -1 & 0 & +1 & +1 \end{vmatrix} \times \begin{vmatrix} I_{-4} \\ I_{-3} \\ \vdots \\ I_0 \\ \vdots \\ I_{+4} \end{vmatrix}$$

We need only pre-multiply both sides of the last equation by the line vector of the b_{qn}s required in order to get the desired result C_{11}. Hence the line vector which generates directly this result from the column vector of the I's is obtained if we form the product

$$\tfrac{1}{2} \begin{vmatrix} -1 & 0 & +1 \end{vmatrix} \times \tfrac{1}{3} \begin{vmatrix} -1 & -1 & 0 & +1 & +1 & 0 & 0 \\ 0 & -1 & -1 & 0 & +1 & +1 & 0 \\ 0 & 0 & -1 & -1 & 0 & +1 & +1 \end{vmatrix} \qquad \ldots \quad (8)$$

$$= \tfrac{1}{6} \times \begin{vmatrix} +1 & +1 & -1 & -2 & -1 & +1 & +1 \end{vmatrix}$$

$$= \sum_n \sum_m b_{qn} \,.\, a_{pm} \,.\, I_{m+n} = \sum_{k=-M-N}^{k=+M+N} r_k \,.\, I_k$$

where $k = m + n$ and where r_k denotes all possible products $b_{qn} \,.\, a_{pm}$ in which the values m and n add up to k. These products are easily formed if we write down on paper the line of coefficients a_{pm}, and then on a strip of paper in reverse order the line of coefficients b_{qn}; we need then only multiply each coefficient of B by that of A standing underneath; k equals the distance between the central term of the a's and the central term of the b's. Thus, for instance, in the case where $p = 1$, $q = 1$, $A = 2$ and $B = 1$ the calculation of the coefficient $r_k = r_{-1}$ looks as follows:

$$\tfrac{1}{2} \times \boxed{+1 \quad 0 \quad -1}$$

$$\tfrac{1}{3} \times \boxed{-1 \quad -1 \quad 0 \quad +1 \quad +}$$

This line of reasoning can be easily generalized so as to cover the case of more than two input frequencies. Suppose the input e.m.f. contains a third frequency of amplitude s. We need then only provide a current (column) vector comprising $2s$ more values than we had before and extend the first matrix by which we multiplied it (the a — matrix) by $2s$ of those staggered lines; we proceed similarly with the b-matrix. The result of the multiplication will be a column vector of values $C_{11}(-s), C_{11}(-s+1) \ldots C_{11}(+s)$ and we need only pre-multiply this by the line-vector corresponding to the third modulation component in order to get again a matrix consisting of a single line, giving us the desired series of coefficients r_k for this case.

It is perhaps of interest to discuss shortly the relation between the formulae developed here and those which were derived recently

by the author in another paper.* The most obvious difference is, of course, that the present formulae save the work involved in the formation of the difference table. On the other hand the formulae given here deal only with the case of integral amplitudes of the grid swing, while the difference method is not limited in this way. It is easily seen, however, that this restriction is purely accidental and only due to the fact that the known formulae for the calculation of harmonic output of the type (2) deal only with the case of integral amplitudes. If formulae of this type are available dealing with non-integral amplitudes (such formulae have recently been derived by the author) the same line of reasoning allows the derivation of the corresponding formulae for intermodulation products.

A difference of a more subtle nature, however, still exists: it has to do with the way in which the analysis underlying the formulae "draws" a smooth curve through the given current ordinates. In the present method we always take groups of m given current ordinates and—using the known formulae of the type (2)—we assume them to be linked by a parabola of the $(m-1)$th order. There are n sets of m points, and n parabolae of the kind mentioned. These parabolae may all be parts of one and the same parabola of the $(m-1)$th order in the case where such a parabola can be passed through all the ordinates given. In general a parabola of higher order is required to fulfil this condition. The difference method uses such a parabola and thus possesses the advantage of (theoretically) higher accuracy. The difference in practice is slight and the question which of these methods gives a closer approach to the actual valve characteristic is a debatable one: the higher order terms which we have to include in the expression for the parabola in order to make it pass exactly through all the given points may impart to this curve a "waviness" which is not present in the actual characteristic. We may, of course, have on occasion to deal with data which do not justify any further smoothing; i.e., which demand the use of the higher order parabola. For this reason it may be of interest to mention that it is possible to derive from the difference method formulae of the type (6), so that even in this case the actual amount of arithmetical work can be cut down to a minimum.*

* "Calculation of Intermodulation Products by Means of a Difference Table," *Journal Inst. El. Eng.*, Vol. 93, Part III, No. 23, p. 211.

* "Lagrangean Formulae for the Direct Calculation of Harmonic Output and of Intermodulation Products." Accepted for publication in the *Philosophical Magazine*. These formulae cover also the case of non-integral amplitudes.

APPENDIX

COEFFICIENTS a_{pm} FOR OUTPUT $p\omega_a$

To facilitate the derivation of formulae for other combination frequencies we reproduce here the coefficients derived in Espley's paper for the calculation of harmonic output (see reference footnote ‡) They are arranged in Table form subject to the same conventions as used above in Tables I and II.

Amplitude	P	Common Factor	−5	−4	−3	−2	−1	0	+1	+2	+3	+4	+5
1	0	1/4					+ 1	+ 2	+ 1				
	1	1/2					− 1	0	+ 1				
	2	1/4					+ 1	− 2	+ 1				
2	0	1/6				+ 1	+ 2	0	+ 2	+ 1			
	1	1/3				− 1	− 1	0	+ 1	+ 1			
	2	1/4				+ 1	0	− 2	0	+ 1			
	3	1/6				− 1	+ 2	0	− 2	+ 1			
	4	1/12				+ 1	− 4	+ 6	− 4	+ 1			
3	0	1/1280			+ 167	+ 378	− 135	+ 460	− 135	+ 378	+ 167		
	1	1/640			− 167	− 252	+ 45	0	− 45	+ 252	+ 167		
	2	1/2560			+ 559	+ 486	− 1215	+ 340	− 1215	+ 486	+ 559		
	3	1/256			− 45	+ 36	+ 63	0	− 63	− 36	+ 45		
	4	9/1280			+ 17	− 42	+ 15	+ 20	+ 15	− 42	+ 17		
	5	81/1280			− 1	+ 4	− 5	0	+ 5	− 4	+ 1		
	6	81/2560			+ 1	− 6	+ 15	− 20	+ 15	− 6	+ 1		

CORRESPONDENCE

Letters of technical interest are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

Iron-Dust Cores

To the Editor, "Wireless Engineer."

SIR,—I was rather surprised to discover, on reading your note on p. 156 of the June issue, that it is still apparently not generally recognized that the permeability formula quoted by Mr. Burgess, from certain of the papers referred to in the Radio Research Board Report of 1934 (with which your result of 1933 is, as you remark, sensibly identical), is probably about a century old. The problem is identical with that covered by Poisson's Theory of Magnetisation, and the formula is given in Maxwell's "Treatise," Vol. 2, ¶ 430, and Vol. 1, ¶ 314 (pp. 57 and 440, respectively, 3rd edition).

In view of the historical interest of this formula, and in order to avoid confusion in the future, it seems desirable to have some means of indicating its classical origin. The related result in electrostatics is usually associated, in one form or another, with the names of Clausius and Mosotti, so there would be a precedent for adopting this description in magnetostatics. Since, however, the result for the electrostatic problem was originally derived from that for the magnetostatic one, such a procedure would be, perhaps, undesirable. I would suggest, as an alternative, that the result be referred to as the "Poisson-Maxwell" formula.

I noticed the identity of these various formulæ 2–3 years ago, but as the result itself is not of great practical value, I did not, at the time, think the point worthy of mention.

Incidentally, it is possible to derive the result for the simple cubic model by methods other than that given in your note of 1933.

ALAN FAIRWEATHER.

Physics Group,
P.O. Research Station,
Dollis Hill, N.W.2.

Pulse Modulation

To the Editor, Wireless Engineer.

SIR,—We have read with interest the article by Roberts and Simmonds in the November issue of *Wireless Engineer* (Vol. 22, p. 538), and the criticisms of Section 1.4 put forward by Mr. R. B. Shepherd in the April issue (Vol. 23, p. 114). We agree with Mr. Shepherd that the derivations in the article are pseudo-static so that they should be treated only as approximations.

There are some inconsistencies in the article between the definitions of types of pulse modulation in the technical description and those implied in the analysis.

In the case of pulse-amplitude modulation the formula derived for flat-topped pulses (Fig. 4) actually applies to a form of amplitude modulation, which may be called "top-modulation," in which the top of a pulse is modulated at each instant by the modulating voltage present at that instant. Such pulses may be regarded as vertical slices of the modulating waveform. Incidentally, the methods of producing amplitude modulation described in 1.3.1. would give this form of pulse amplitude modulation and not the rectangular pulses shown in Fig. 4.

The spectrum implied in (6) does not correspond to length-modulated pulses whose centres occur at fixed time intervals, as suggested in 1.4.2. It does correspond, however, to the method of modulation described in 1.3.2. When using this method, the centre of the pulse (which we take to be midway between the two edges) does not recur at equal intervals of time.

The waveform of phase-modulated pulses given in (7), obtained as in 1.4.3., certainly contains no term at the modulating frequency, but it does not conform with the commonly accepted definition given in 1.3.3. which specifies that the phase-modulated pulses retain constant length. It can be shown that the pulses described by (7) suffer length, as well as phase, modulation such that the component at the modulating frequency vanishes.

Turning to Mr. Shepherd's suggestions for a more exact study, it appears that his treatment of pulse-amplitude modulation, though quite different from that developed by us, gave an identical result. He bases his method on the assertion that, in all three types of pulse modulation, amplitude, length and phase, "the property of the pulse chain which is being varied is determined by the modulating signal at a number of instants equally spaced along the time-axis." While this is true for flat-topped amplitude modulation, it is not true for length or phase modulation where the modulation process depends on the slicing of a sawtooth voltage at a variable level. If such a process is examined, it will be seen that each edge subjected to independent modulation is shifted in time by an amount which is proportional to the modulating voltage at the instant when the edge occurs. Clearly, these instants are not uniformly spaced, and so Mr. Shepherd's assertion cannot be true. This renders his treatment of length and phase modulation approximate also.

An exact investigation of the modulation, made at S.R.D.E., leads to a formula for length-modulated pulses identical with (6); the formula for phase-modulated pulses of constant length is

$$i = i_{\max} S + i_{\max} \frac{Sp}{\omega} \phi_{\max} \left\{ \frac{\sin \pi Sp/\omega}{\pi Sp/\omega} \right\} \cos pt + i_{\max} S \sum_{k=1}^{\infty} \sum_{n=-\infty}^{\infty} J_n(k\psi_{\max}) \frac{\sin \pi S(k + n\omega/p)}{\pi S(k + n\omega/p)} \cos (k\omega + np)t$$

in the notation of Roberts and Simmonds. The term at the modulating frequency, $p/2\pi$, differs from that given by Mr. Shepherd by the factor in curly brackets. In most applications S will be small; this factor is then nearly unity and Mr. Shepherd's formula gives a good approximation.

A theoretical study of the spectrum of modulated pulses for a number of typical modulation systems

has been made at S.R.D.E. Results were obtained without approximation, and a report with limited circulation was issued in September, 1945; it is hoped that it will be published shortly.

EDGAR FITCH.

Signals Research and Development Establishment, Christchurch.

To the Editor, "Wireless Engineer"

SIR,—Some conclusions which have been reached pertaining to Section 1.4 of the article by F. F. Roberts and J. C. Simmonds[1] and to R. B. Shepherd's criticism,[2] recently published in your journal are submitted below.

Let us consider the spectrum analysis of width-modulated pulses. Although I am not in disagreement with Mr. Shepherd's criticism of Messrs. Roberts' and Simmonds' method of analysis, I should like to point out that their method is entirely correct under certain conditions. The exact law governing the modulation must be taken into account and it should be noted that many such laws are possible. In the above-mentioned articles the instants at which pulses start and stop are determined by the signal amplitude at fixed time intervals. If the instants of the voltage jumps are determined by the signal amplitudes at the times when those jumps occur, the mathematical expression turns out to be simpler and the analysis indicated by Messrs. Roberts and Simmonds applies exactly. A proof follows at the end of this communication.

The law of modulation just defined is easily realized physically by adding to the signal voltage a saw-tooth voltage of larger amplitude, clipping, and then limiting the resultant.[3] The methods of W. A. Beatty[4] also yield pulses whose width modulation is governed by this law. The mathematical expression, placing in evidence the individual components of the spectrum, is

$$(1)\quad e = S_0(1 - A\cos pt) + 2/\pi \sum_{\nu=1}^{\infty}\sum_{n=-\infty}^{\infty} 1/\nu \left\{ J_{(n)}\left[\frac{Ad_0}{2}(\nu\omega)\right] \sin\left[\frac{d_0}{2}(\nu\omega) - \frac{n\pi}{2}\right]\right\} \cos(\nu\omega + np)t$$

where ω = pulse-repetition frequency in angular measure.

p = modulating-signal frequency in angular measure.

d_0 = average pulse duration.

$S_0 = \frac{\omega d_0}{2\pi}$ = average duty cycle.

A = modulation index.

This expression is for "symmetrical" width modulation by a sinusoidal signal, although the pulses do not generally widen and narrow symmetrically with this modulation law, when the modulating frequency is near the pulse repetition frequency. For symmetrical pulse-width modulation in accordance with the law specified by the above-named authors, the corresponding spectrum is[5].

$$(2)\quad e = \sum_{\nu=0}^{\infty}\sum_{n=-\infty}^{\infty} \frac{1}{\nu + n/p\omega}\left\{ J_{(n)}\left[\frac{Ad_0}{2}(\nu\omega + np)\right] \sin\left[\frac{d_0}{2}(\nu\omega + np) - \frac{n\pi}{2}\right]\right\} \cos(\nu\omega + np)t$$

Similar analyses exist, of course, for asymmetrically width-modulated pulses, in which one pulse edge remains fixed and the other edge moves in accordance with the law specified.

There are several noticeable differences between the two spectra given by (1) and (2) which are of interest from the point of view of audio fidelity. It should be noted that these considerations also apply to pulse-time or delay modulation systems in which width-modulated pulses are produced in the receiver for detection purposes.

The proof that the instants of voltage jumps are determined by the signal amplitudes at those instants is as follows:—

By ordinary Fourier analysis it is found that the series

$$e = S + 2/\pi \sum_{\nu=1}^{\infty} [1/\nu \sin(\pi S)] \cos \nu\omega t.$$

represents the pulse train shown in Fig. 1.

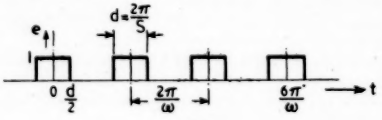


Fig. 1.

If one lets $\theta = \omega t$, the function becomes

$$e(S, \theta) = S + 2/\pi \sum_{\nu=1}^{\infty} [1/\nu \sin(\pi S)] \cos \nu\theta$$

This function equals either one or zero for any value of θ provided $0 < S < 1$. It is a simple matter to map the regions of the $S - \theta$ plane in which the function has the value one. This is shown in Fig. 2, which is correct regardless of the

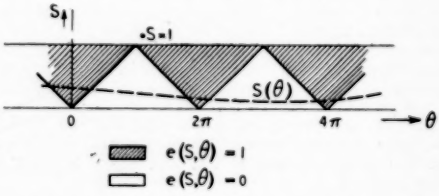


Fig. 2.

manner in which S and θ may vary. In particular, S may be a function of θ, as indicated by the broken line. When the line lies in the shaded regions, the

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[1] Roberts, F. F., and Simmonds, J. C., "Multichannel Communications Systems, Preliminary Investigation based upon Modulated Pulses," *Wireless Engineer*, Vol. 22, No. 266, p. 538, November 1945.

[2] Shepherd, R. B., Letter to the Editor, *Wireless Engineer*, Vol. 23, No. 271, p. 114, April, 1946.

[3] Kell, R. D., U.S. Patent 2,061,734.
Beatty, W. A., British Patent 523,575.

[5] Fredendall, G. L.; Schlesinger, K.; and Schroeder, A. C., "Transmission of Sound on the Picture Carrier," *Proc. I.R.E.*, Vol. 34, pp. 49–61, February, 1946.

value of $e(S, \theta)$ is unity, otherwise, it is zero. Since θ is the time angle and S the relative pulse which is to be varied in accordance with a signal, the instant at which the transition occurs depends upon the signal amplitude at that instant.

It is of interest to notice that the saw-tooth boundary and the dotted line are directly related to the corresponding quantities in the physical modulation process.

E. R. KRETZMER.

Massachusetts Institute of Technology,
Cambridge, Mass., U.S.A.

NEW BOOKS

Alternating Current Measurements at Audio and Radio Frequencies (2nd Edition).

By DAVID OWEN, B.A., D.Sc., F.Inst.P. Pp. 120 + vii, with 80 diagrams. Methuen and Co., Ltd., 36, Essex Street, London, W.C.2. Price 5s.

The author succeeded so well, by his selection and presentation of material in the first edition of this little book, in providing in compact form most of the really useful information that one would expect to find in a far larger volume, that one can understand any reluctance to make any very drastic changes in preparing the second edition, now published. Revision has been confined for the most part to such slight improvements in the text as there was any need to make. As regards symbols, replacement of " p " by " ω " is in line with present-day standards.

The author's deliberate departure from the standard notation for vector quantities is more questionable, however. It will be granted that the prescribed **I, V** and **Z** in Clarendon type, corresponding to the scalar I, V and Z, are badly suited to purely manuscript work. But they were never intended for it ; the standard MS. notation—a bar over the letter—is convenient to use and less likely to be misread than Dr. Owen's z for the impedance operator. If the MS. is intended for printing, the standard means of indicating bold type—$\underline{Z}$, etc.—is quite practicable and unambiguous. Dr. Owen's $\mathcal{I}$, $\mathcal{V}$, and $\mathcal{E}$ are little if any easier to write than $\bar{I}$, $\bar{V}$, and $\bar{E}$, and are more likely to be confused in typesetting ; in any case $\mathcal{E}$ is already the standard symbol for electric field strength. The reviewer does not agree that in printed work there is any justification for abandoning **I, V** and **Z,** least of all on the ground of difficulty in distinguishing them from the corresponding scalar symbols. In a printed page they are the most easily picked out of any.

The chapter on r.f. measurements, occupying about one-third of the book, seemed even in 1937 to be rather less well abreast of current practice than the a.f. chapters, and as in spite of relatively rapid advances in practice this chapter is substantially unchanged in the second edition, it contains features that are now decidedly outmoded. Among them is Fig. 55, a circuit diagram of a wavemeter showing terminals for an external coupling coil directly in the calibrated tuned circuit, a variable capacitor across phones and H.T., and sliding short-circuit contacts on reaction coil and main tuning coil, but no grid bias or capacitor.

The author's sound, clear and concise presentation and his avoidance of anything like padding, make his work so helpful to the busy reader that it is to be hoped that in a future edition he will bring the r.f. chapter more completely up to date. The reviewer suggests, too, that the Twin-T method of reactance and resistance measurement, which combines the advantages of null methods with that of having a common earthy point for source, detector and unknown, deserves admission.

M. G. S.

Theorie des Oscillateurs

By YVES ROCARD. Pp. 223 + viii and 75 Figs.

This book is a wartime product, written in 1940 and published in 1941, and the author states this as a reason for the absence of a bibliography, but refers readers to the papers by van der Pol in the Philosophical Magazine between 1922 and 1928, and to one by Liénard in the Revue générale d'Electricité of May 26 and June 2, 1928. The author deals with many different types of oscillations and goes into them very thoroughly. Chapter I is entitled the pendulum, but before coming to the real pendulum, electric circuits and vibrating cantilevers are considered ; the pendulum is analysed very thoroughly, including the electrically maintained pendulum. Chapter II deals with relaxation oscillations, synchronization, production of submultiples of the frequency, *Ziehen*, and allied phenomena. In the following chapter oscillators without self-inductance or inertia are considered. Chapter IV deals with a complicated type of oscillation called " non-holonomes " and three-wheeled chariots with spring-controlled steering. Chapter V leaves engineering and deals with economic oscillations ; this method of considering economic crises was introduced by Kalecki in 1933. The following chapter is concerned with electromagnetic and acoustic propagation in so far as they are involved in oscillatory systems such as resonant lines and pipes. The singing arc and allied phenomena are considered in Chapter VII, and the hunting of rotating machines in Chapter VIII. The final chapter deals with the important subject of the vibration of the wings of aeroplanes.

This book can be heartily recommended to anyone seriously interested in the subject of oscillations.

G. W. O. H.

Whittaker's Electrical Engineers' Pocket Book (7th Edition).

Edited by R. E. NEALE, B.Sc., A.C.G.I., A.M.I.E.E. Pp. 938 + ix. Published by Sir Isaac Pitman & Sons Ltd., Pitman House, Parker Street, Kingsway, London, W.C.2. Price 30s.

Cours de Radioélectricité Générale. Tome II, Les Lampes Amplificatrices.

By P. DAVID. Pp. 206. Published by Librairie de l'Enseignement Technique, 61, Boulevard Saint-Germain, Paris. Price 370 francs.

BACK ISSUES

Copies of *Wireless Engineer* for March, 1946, are required by Public and University Libraries in the U.S.A. Readers who have copies for disposal are asked to communicate with the Publisher.

WIRELESS PATENTS

A Summary of Recently Accepted Specifications

The following abstracts are prepared, with the permission of the Controller of H.M. Stationery Office, from Specifications obtainable at the Patent Office, 25, Southampton Buildings, London, W.C.2, price 1/- each.

ACOUSTICS AND AUDIO-FREQUENCY CIRCUITS AND APPARATUS

574 726.—Generating and utilizing short pulses of sound, for calibrating loudspeakers and microphones, and for measuring the acoustic properties of studios and concert halls.

Western Electric Co., Inc. Convention date (U.S.A. 25th July, 1942.

574 749.—Microphone-loudspeaker installation for measuring the acoustic properties of a room by increasing the relative effect or " weight " of the fading echoes of " testing " sounds.

Western Electric Co., Inc. Convention date (U.S.A.) 17th November, 1942.

574 808.—Back-coupled valve equipment for detecting and locating land-mines, booby-traps, and other metallic objects.

Cinema-Television, Ltd., and S. S. West. Application dates 21st May and 10th September, 1943.

574 941.—Oscillator and feed-back circuit for indicating the amount of unbalance in an impedance network and, more particularly, for locating buried conductors.

Hazeltine Corporation (assignees of L. F. Curtis) Convention date (U.S.A. 31st December, 1941.

574 955.—Balanced inductor system, associated with an oscillation generator and a receiver, for detecting the presence of buried conductors.

Hazeltine Corporation (assignees of H. A. Wheeler). Convention date (U.S.A.) 31st December, 1942.

DIRECTIONAL WIRELESS

574 710.—Cathode-ray indicator for a radio-compass in which the pick-up voltages from directional and " sense " aerials are periodically switched across each pair of deflecting-plates.

Marconi's W.T. Co., Ltd., and N. H. Clough. Application date 16th October, 1940.

574 933.—Blind-landing system in which the glide-path is continuously indicated by a radio-altimeter which compares the normal ground-signals with those received from a reflector on the landing field.

Standard Telephones and Cables, Ltd., C. W Earp, and C. E. Strong. Application date 2nd April, 1940.

574 950.—Radiolocation or other amplifier utilizing a number of narrow band-width stages, in which provision is made for the rapid damping of undesired oscillations during periodical switching operations.

W. L. Watton. Application date 23rd November, 1943.

574 964.—Frequency meter designed also to interpret the effect of sudden changes in phase, particularly in radiolocation equipment of the frequency-modulated type.

D. Weighton and Pye, Ltd. Application date 9th September, 1940.

574 965.—Radio-altimeter of the frequency-modulated type in which the reflected signal is converted into an amplitude-modulated equivalent in order to give low-altitude indications.

D. I. Lawson and Pye, Ltd. Application date 9th September, 1940.

RECEIVING CIRCUITS AND APPARATUS

574 626.—Resistance-capacitance coupling designed to offset the stray capacitances at the grid of the second valve in a two-stage amplifier.

Standard Telephones and Cables, Ltd., C. T. Scully and A. Waters. Application date 11th February, 1944.

574 811.—Capacitance controlled switch for correcting the " overshoot " due to inertia, say in the tuning control of a radio receiver which is constantly swept through a band of frequencies and is required to stop on receipt of a given signal.

E. L. C. White, M. G. Harker and J. P. W. Houchin. Application date 10th June, 1943.

574 823.—Amplifier with a double feed-back circuit for increasing the signal-to-noise ratio, particularly in the frequency-band covered by the A.C. mains-supply and its harmonics.

Standard Telephones and Cables, Ltd. (assignees of J. O. Weldon). Convention date (U.S.A.) 25th November, 1942.

TRANSMITTING CIRCUITS AND APPARATUS

574 838.—Blocking-oscillator circuit which generates a pulsed output rich in harmonic frequencies, suitable for calibration purposes or for multi-channel signalling.

Hazeltine Corporation (assignees of R. E. Sturm). Convention date (U.S.A.) 20th September, 1941.

574 932.—Combined short-wave transmitting and receiving set designed to be concealed about the person of the operator.

The British Thomson-Houston Co., Ltd. and T. H. Kinman. Application date 30th January, 1940.

SIGNALLING SYSTEMS OF DISTINCTIVE TYPE

574 769.—Time-constant circuit for reducing the effect of " backlash " voltage in a multivibrator which generates timed pulses in response to a triggering signal.

E. L. C. White and F. C. Williams. Application date 29th July, 1942.

574 844.—Generating pulses, suitable for echo-ranging, by periodically applying intense light-flashes to the photo-sensitive cathode of a magnetron or other type of oscillator.

The British Thomson-Houston Co., Ltd. (communicated by General Electric Co.). Application date 27th May, 1943.